POORNIMA
COLLEGE OF ENGINEERING

TUTORIAL SHEET

TUTORIAL SHEET — SHEET No. 5

Campus: P.C.E. Course: B.Tech Class/Section: II 4 Year Date: 30/8/19

Name of Faculty: Dr. Shilpi Jain Name of Subject: AEM Code: 3CE2-01

Date of Tut. Sheet Preparation:........ Scheduled Date of Tut.:........ Actual Date of Tut.:........

Name of Student:........ Scheduled & Actual Date of H.A. Submission:........&........

	Questions	CO	PO
FIRST 20 MT. CLASS QUESTIONS	Q.1. Obtain the Laplace transform of $\frac{\cos at-\cos bt}{t}$.	4	2
	Q.2. (a) Evaluate $L\left\{\frac{e^{-3t}\sin 2t}{t}\right\}$ (b) $\left\{\frac{\cos 2t-\cos 3t}{t}\right\}$	4	2
2 HRS. SOLVABLE HOME ASSIGNMENT (H.A.) QUESTIONS	Q.3. find the $L\left\{e^{5t}\cos 9t\right\}$	1	1
	Q.4. Obtain (a) $L\left\{\frac{e^{-t}\sin t}{t}\right\}$ (b) $L\left\{\int_0^\infty t e^{-3t}\sin 2t\, dt\right\}$	4	2
OTHER IMPORTANT QUESTIONS	Q.5. Using Convolution Th., Solve (a) $L^{-1}\left\{\frac{s}{(s^2+1)^2}\right\}$ (b) $L^{-1}\left\{\frac{1}{(s^2-4)^2}\right\}$	4	2

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Solution

Soln 1. $L\left\{\frac{\cos at-\cos bt}{t}\right\} = \int_s^\infty \left(\frac{s}{s^2+a^2} - \frac{s}{s^2+b^2}\right)ds = \left[\frac{1}{2}\log\frac{s^2+a^2}{s^2+b^2}\right]_s^\infty$

$= \frac{1}{2}\log\frac{s^2+b^2}{s^2+a^2}$

Soln 2. (a) $L\left\{\frac{e^{-3t}\sin 2t}{t}\right\} = \int_s^\infty \left[\frac{2}{(s+3)^2+4}\right] ds = 2\left[\frac{1}{2}\tan^{-1}\frac{(s+3)}{2}\right]_s^\infty$

$= \tan^{-1}\infty - \tan^{-1}\frac{s+3}{2}$

$= -\cot^{-1}\frac{s+3}{2}$

(b) $L\left\{\frac{(\cos 2t-\cos 3t)}{t}\right\} = \frac{1}{2}\log\frac{s^2+9}{s^2+4}$

Soln 3 $L\{e^{5t}\cos 9t\} = \left[\frac{s-5}{(s-5)^2+3^2}\right]$ [$\because$ using Ist shift Rule]

Soln (4) (a) $L\left\{e^{-t}\int_0^t \frac{\sin t}{t}\,dt\right\}$

Let $f(t) = \frac{\sin t}{t}$

$L\{f(t)\} = \int_s^\infty \frac{1}{s^2+1}ds = [\tan^{-1}s]_s^\infty = \cot^{-1}s$

$\Rightarrow L\left\{\frac{e^{-t}\sin t}{t}\right\} = \cot^{-1}(s+1)$

(b) $L\left\{\int_0^\infty t\,e^{-3t}\sin 2t\,dt\right\} = (-1)^1\frac{d}{ds}\left[\frac{2}{(s+3)^2+4}\right]$

$= -\frac{d}{ds}\left[(s+3)^2+4\right]^{-1}$

$= \left[(s+3)^2+4\right]^{-2}\cdot 2(s+3)$

$= \frac{2(s+3)}{\left[(s+3)^2+4\right]^2}$

Taking $s\to 0$ both sides.

Now $\int_0^\infty t\,e^{-3t}\sin 2t\,dt = \frac{6}{[9+4]^2} = \frac{6}{(13)^2}$

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Q.5. (a) $L^{-1}\left\{\frac{s}{(s^2+1)^2}\right\}$

Conv. Th.

$$= \frac{2}{2}\int_0^t \cos u \cos(t-u)\,du$$

$$= \frac{1}{2}\left[\int_0^t [\cos(\cancel{u}+t-\cancel{u}) + \cos(u-t+u)]\,du\right.$$

$$= \frac{1}{2}\left[\cos t\,[u]_0^t + \left[\frac{\sin(2u-t)}{2}\right]_0^t\right]$$

$$= \frac{1}{2}\left[t\cos t + \frac{\sin t}{2}\right] + \frac{\sin t}{2}\Bigg] = \frac{1}{2}[t\cos t + \sin t]$$

Soln. 5.(b) $L^{-1}\left\{\frac{1}{(s^2+4)^2}\right\}$

Conv. Th.

$$= \frac{2}{2}\int_0^t \sin 2u \sin 2(t-u)\,du$$

$$= \frac{1}{2}\left[\int_0^t [\cos(2u-2t+2u) - \cos(\cancel{2u}+2t-\cancel{2u})]\,du\right.$$

$$= \frac{1}{2}\left[\frac{\sin(4u-2t)}{4} - (\cos 2t)\,u\right]_0^t$$

$$= \frac{1}{2}\left[\frac{\sin 2t}{4} - t\cos 2t + \frac{\sin 2t}{4}\right]$$

$$= \frac{1}{2}\left[\frac{\sin 2t}{2} - t\cos 2t\right]$$

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