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2.3.1 Tutorial Sheets (Sample)
(2023-24)

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ADVANCED ENGG. MATHS 3ed sem(IT)

TUTORIAL SHEET–II

UNIT-II (PROBABILITY DISTRIBUTION)

<u>NOTE:-</u> Attempt all questions.

Q.(1) Find mean, variance and standard deviation of Binomial Distribution.

Q.(2) In a certain factory turning out razor blades there is a small chance of 0.002 for any blade to be Defective .the blades are supplied in a packet of 10. Using Poisson distribution, find the Number of packets containing no defective, one defective, and two defective blades respectively in a consignment of 10,000 packets.

Q.(3) The distribution of weekly wages for 500 workers in a factory is approximately normal with The mean and standard deviation of Rs.75 and Rs.15 respectively .find the number of workers Who receive weekly wages (i) more than Rs.90 (ii) less than Rs.46. (given $p(0 \leq z < 1) = 0.3413$,$P(0<z<2) = 0.4772$)

Q.(4) Fit a Binomial distribution to the following data :-

x	0	1	2	3	4	5
f	2	14	20	34	22	8

Q.(5) Find the Karl Pearson coefficient of correlation from the following data.

x	25	27	30	35	33	28	36
y	19	22	27	28	30	23	28

Q.(6) The ranks of 10 students in two subjects mathematics and physics are given below:

Ranks in mathematics x	5	2	9	8	1	10	3	4	6	7
Ranks in physics y	10	5	1	3	8	6	2	7	9	4

Calculate Spearman’s Rank Correlation coefficient

Q.(7) Calculate the coefficient of correlation and obtain the line of regression for the following data.

x	1	2	3	4	5	6	7	8	9
y	9	8	10	12	11	13	14	16	15

Q.(8) Define Exponential distribution? Find the mean and variance of the distribution.

Q.(9) Fit the data in second degree curve (parabola) by method of least square.

x	0	1	2	3	4
y	1	1.8	1.3	2.5	6.3

Q.10 Prove that the angle between two regression lines is $\theta = tan^{-1}\{(\frac{1-r^2}{r})\frac{\sigma x \sigma x}{\sigma_x{}^2+\sigma_y{}^2}\}$ where r is the Correlation coefficient.

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Unit II - Binomial Distribution

Q1) Find mean, variance and standard deviation of binomial distribution.

Mean

Let x be any discrete random variable

$\mu_1' = E(x) = \Sigma x P(x)$

$P(r) = {}^nC_r p^r q^{n-r}$ $[r = 1, 2, 3, \ldots n]$

$\mu_1' = \sum_{r=1}^{n} r \cdot {}^nC_r p^r q^{n-r}$.

$\mu_1' = {}^nC_1 pq^{n-1} + 2\,{}^nC_2 p^2 q^{n-2} + 3\,{}^nC_3 p^2 q^{n-3} + \ldots$

$= npq^{n-1} + 2(n)(n-1)p^2q^{n-2} + 3(n)(n-1)(n-2)p^3q^{n-3} + \ldots$

$\mu_1' = npq^{n-1}\left[1 + (n-1)\frac{p}{q} + (n-1)(n-2)\frac{p^2}{q^2}\right]$

OR

$\mu_1' = E(x) = \sum_{r=0}^{\infty} r \cdot {}^nC_r p^r q^{n-r}$

$= \sum_{r=0}^{\infty} r \frac{\lfloor n}{\lfloor r \lfloor n-r} p^r q^{n-r}$.

$\sum_{r=0}^{\infty} \frac{r \lfloor n}{r \lfloor r-1 \lfloor n-r} p^r q^{n-r}$

$\sum_{r=0}^{\infty} \frac{n \lfloor n-1}{\lfloor r-1 \lfloor n-1-r+1} \times p^{r-1+1} q^{n-1-r+1}$

$= \sum_{r=1}^{\infty} np \frac{\lfloor n-1}{\lfloor r-1 \lfloor n-1-r+1} p^{r-1} q^{n-1-r+1}$

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$$= \sum_{r=1}^{\infty} np \, {}^{n-1}C_{r-1} \, p^{r-1} \, q^{n-1-r+1}$$

$$= np(p+q)^{n-1} = np$$

So $\eta = \mu_1' = np.$

Variance.

$$\sigma^2 = \mu_2 = E(X^2) - [E(X)]^2$$

$$\mu_2' - (\mu_1)^2 \quad \text{— (1)}$$

$$\mu_2' = E(X^2) = \sum_{r=0}^{\infty} r^2 p(r)$$

$$\sum_{r=0}^{\infty} r^2 \, {}^nC_r \, p^r q^{n-r}.$$

$$= \sum_{r=0}^{\infty} [r(r-1)+r] \, {}^nC_r \, p^r q^{n-r}.$$

$$\sum_{r=0}^{\infty} r(r-1) \, {}^nC_r \, p^r q^{n-r} + \sum_{r=0}^{\infty} r \, {}^nC_r \, p^r q^{n-r}$$

$$= \sum_{r=0}^{\infty} r(r-1) \frac{\lfloor n}{\lfloor r \lfloor n-r} p^r q^{n-r} + np$$

$$= \sum_{r=0}^{\infty} \frac{r(r-1) \lfloor n}{r(r-1) \lfloor r-2 \lfloor n-r} p^r q^{n-r} + np$$

$$\sum_{r=2}^{\infty} \frac{n(n-1) \lfloor n-2}{\lfloor r-2 \lfloor n-2-r+2} p^{r-2+2} \cdot q^{n-2-r+2} + np.$$

$$= n(n-1)p^2 (p+q)^{n-2} + np.$$

$$\mu_2' = n(n-1)p^2 + np$$

$\mu_2' = n^2p^2 - np^2 + np$ — (2)

$\sigma^2 = n^2p^2 - np^2 + np - n^2p^2$

$\sigma^2 = np - np^2$

$\sigma^2 = np(1-p) = npq$

$\sigma^2 = npq$ — variance

$SD = \sigma = \sqrt{\text{variance}}$

so standard deviation $= \sqrt{npq}$

Q2) In a certain factory turning out razor blades there is a small chance of 0.002 for any blade to be defective the blades are supplied in a packet of 10. using poison distribution, find the number of packets containing no defective, one defective, and two defective blades respectively in a consignment of 10,000 packets

Ans given $p = 0.002$

$n = 10$

$M = 10,000$

given condition

(i) No defective

(ii) one defective

(iii) ~~two~~ defective

Mean $= m = np = 10 \times 0.002 = 0.02$

we have by poison's distribution

$p(x=r) = \dfrac{e^{-m} m^r}{\lfloor r}$; $r = 0, 1, 2, \ldots$

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(i) $p(x=0) = \dfrac{e^{-0.02} \times (0.02)^0}{\underline{|0}} = 0.9801$

So No of packets having no defective blades out of 10,000 packets

$f(x=0) = N \times P(x=0)$

$= 10,000 \times 0.9801 = 9801$ packets

(ii) $P(x=1) = \dfrac{e^{-0.02} \times (0.02)^1}{\underline{|1}} = 0.0196$

So No of packets having one defective blade out of 10,000 packets

$= 10,000 \times 0.0196 = 196B.$

(iii) $P(x=2) = \dfrac{e^{-0.02}(0.02)^2}{\underline{|2}} = 1.96 \times 10^{-4}.$

So, the no of ~~blades~~ packets having two defective blades out of 10,000 packets.

Q3) The distribution of weekly wages for 500 workers in a factory is approximately normal with the mean and standard deviation of Rs 75 and Rs 15 respectively. Find the number of workers who receive weekly wages

(i) more than Rs 90

(ii) less than Rs 46 (given $p(0 \leq z < 1) = 0.3413$, $P(0 < z < 2) = 0.4772$)

Sol given : $\mu = 75$

$\sigma = 15$

$N = 500$

$Z = \dfrac{x - \mu}{\sigma}$

(i) more than Rs 90

So $x = 90$ $= P(x > 90)$

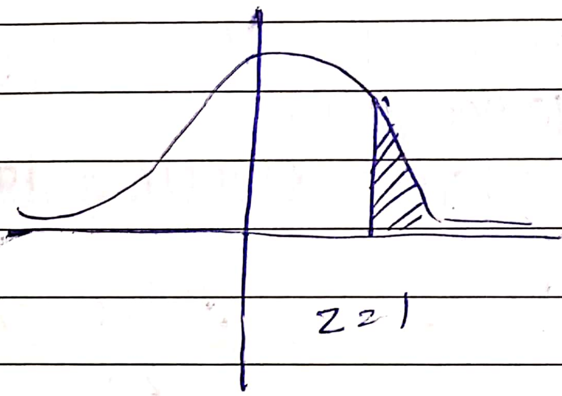
$\dfrac{90 - 75}{15} = 1$

or $P(z > 1)$

$P(0 < z < \infty) - P(0 < z < 1)$

$0.5 - 0.3413$

$= 0.1587$



So the total no of workers who received weekly wages more than Rs 90

$= 0.157 \times 500$

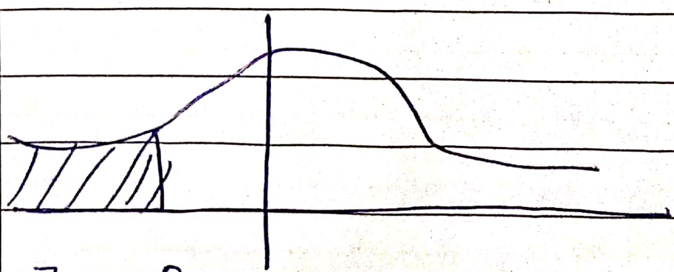
$= 79.35$ Ans :

≈ 79 Ans .

(ii) Less than Rs 46

$P(x < 46)$

$z = \dfrac{46 - 75}{15} = -1.93 = -2$



$z = -2$

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$P(Z < -2) = P(-\infty < Z < 0) - P(-1 < Z < 0)$

$= 0.5 - 0.4772$

$= 0.228$

So, the total numbers of workers who received weekly wages less than Rs 46

$= 0.0228 \times 500$

$= 11.4 \simeq 11$

Q4. Fit a Binomial distribution to the following data

x	0	1	2	3	4	5
f	2	14	20	34	22	8

Ans Here we have f

So $m = \frac{\Sigma x.f}{\Sigma f} = \frac{284}{100} = 2.84$

x	f	x.f
0	2	0
1	14	14
2	20	40
3	34	102
4	22	88
5	8	40

$\Sigma f = 100 \quad \Sigma x.f = 284$

we have

$m = np$

$2.84 = 5(p)$

$p = 0.568$

$q = 1-p = 0.432$

So $= (0.568 + 0.432)^5$

Theoritical frequency

$f(x) = N \times (p+q)^n$

$= 100 \times (0.568 + 0.432)^5$

$= 100 \times 1 + 100 \times 1^2 + 100 \times 1^3 + 100 \times 1^4 + 100 \times 1^5 + 100 \times 1^6 = 600$ ans

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Q5) Find the Kearl Pearson coefficient of carrelation from the following data.

x	25	27	30	35	33	28	36
y	19	22	27	28	30	23	28

Ans We have Kearl Pearson Coefficient:

for $r(x_i, y) = \frac{\Sigma(x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\Sigma(x_i - \bar{x})} \cdot \sqrt{\Sigma(y_i - \bar{y})^2}}$

$A = (x_i - \bar{x})$; $B = (y_i - \bar{y})$

$r(x,y) = \frac{\Sigma AB}{\sqrt{\Sigma A^2}\sqrt{\Sigma B^2}}$

x	y	$x_i - \bar{x}$	$y_i - \bar{y}$	$(x_i - \bar{x})(y_i - \bar{y})$	$(x_i - \bar{x})^2$	$(y - \bar{y})^2$
25	19	-5.57	-6.28	34.97	31.02	39.43
27	22	-3.57	-3.28	11.70	12.74	10.75
30	27	-0.57	1.72	0.98	0.32	3.06
35	28	4.43	2.72	12.04	19.62	7.39
33	30	2.43	4.72	11.46	5.90	22.27
28	23	-2.57	-2.28	5.85	6.60	5.198
36	28	5.43	2.72	14.76	29.48	7.39
				$\Sigma AB - 89.8$	$\Sigma A = 105.68$	$\Sigma B - 95.4$

$\bar{X} = \frac{25+27+30+35+33+28+36}{7} = 30.58$

$\bar{Y} = \frac{19+22+27+28+30+28+28}{7} = 25.29$

$r(x,y) = \frac{89.8}{105.68 \times 95.48} = 8.89 \times 10^{-03}$

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Q6) The ranks of 10 students in two subjects mathematics and physics are given below

Ranks in mathematics x	5	2	9	8	1	10	3	4	6	7
Ranks in physics y	10	5	1	3	8	6	2	7	9	4

Calculate Spearman's Rank Correlation Coefficient

Ans $n = 10$ no of students

given

R_x	R_y	$d = R_x - R_y$	d^2
5	10	-5	25
2	5	-3	9
9	1	8	64
8	3	5	25
1	8	-7	49
10	6	4	16
3	2	1	1
4	7	-3	9
6	9	-3	9
7	4	3	9

Σd^2 216

We have by spearson's Rank correlation formula

$$\rho(x,y) = 1 - \frac{6\Sigma d^2}{n(n^2-1)} = 1 - \frac{6 \times 216}{10 \times 99}$$

$$= -0.3091$$

Hence the rank in subject maths and physics are -ve correlated.

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Q7. Calculate the coefficient of correlation and obtain the line of regression for the following data:

x	1	2	3	4	5	6	7	8	9
y	9	8	10	12	11	13	14	16	15

Ans

X	Y	X-5	Y-12	AB	A^2	B^2
1	8	-4	-4	16	16	16
2	9	-3	-3	9	9	9
3	10	-2	-2	4	4	4
4	12	-1	0	0	1	0
5	11	0	-1	0	0	1
6	13	1	1	1	1	1
7	14	2	2	4	4	4
8	16	3	4	12	9	16
9	15	4	3	12	16	9
				$\Sigma AB = 58$	$\Sigma A^2 = 60$	$\Sigma B^2 = 60$

$$r(x,y) = \frac{\Sigma(x_i-\bar{x})(y_i-\bar{y})}{\sqrt{\Sigma(x_i-\bar{x})^2 \Sigma(y_i-\bar{y})^2}}$$

$(x_i-\bar{x}) = A$; $(y_i-\bar{y}) = B$

$$r(x,y) = \frac{\Sigma AB}{\sqrt{A^2 \Sigma B^2}} = \frac{58}{60 \times 60} = 0.96621$$

So $r(x,y) = 1$

$\sigma x = 0.860$

$\sigma y = 0.60$

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Equation of regression line :-

y on x

$$y-\bar{y} = r\frac{\sigma y}{\sigma x}(x-\bar{x}) \quad -①$$

$$\cancel{y-12=0}$$

$$y-12 = 0.966 \times 1\,(x-5)$$

$$y-12 = 0.966\,x - 4.83$$

$$y = 0.966x + 7.17$$

x on y

$$(x-\bar{x}) = r\left(\frac{\sigma x}{\sigma y}\right)(y-\bar{y})$$

$$x-5 = 0.96 \times \frac{0.860}{0.860}(x-12) = 0.96y - 0.96\times 12$$

$$x = 0.96y - 6.52 \text{ Ans.}$$

Q2 Define Exponential distribution? Find the mean and variance of the distribution.

Ans Exponential Distribution

A continuous random variable X is said to follow an exponents distribution also called as negative exponential distribution with parameter $\lambda>0$ if its probability density function is given by

$$f(x) = \begin{cases} \lambda e^{-\lambda x} & ; x\geq 0 \\ 0 & ; \text{otherwise} \end{cases}$$

The above given is Pdf as $f(x)>0$ and

$$\int_0^\infty f(x)dx = \lambda\int_0^\infty e^{-\lambda x}dx = \lambda\left(\frac{e^{-\lambda x}}{-\lambda}\right)_0^\infty = 1$$

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Mean and Variance

Mean $\mu_1' = \int_0^\infty x \lambda e^{-\lambda x}\, dx$.

$$= \lambda \left[x \cdot \frac{e^{-\lambda x}}{-\lambda} - 1 \cdot \frac{e^{-\lambda x}}{\lambda^2} \right]_0^\infty$$

$$= \lambda \times \frac{1}{\lambda^2} = \frac{1}{\lambda}$$

So $\mu_1' = \frac{1}{\lambda}$

Variance $= \mu_2 = \mu_2' - (\mu_1')^2$

$\mu_2' = \int_0^\infty x^2 \lambda \cdot e^{-\lambda x}\, dx$

$$= \lambda \int_0^\infty x^2 e^{-\lambda x}\, dx$$

$$= \lambda \left[\left(x^2 \cdot \frac{e^{-\lambda x}}{-\lambda} \right)_0^\infty - 2 \int_0^\infty x \frac{e^{-\lambda x}}{-\lambda}\, dx \right]_0^\infty$$

$$= \lambda \left[\underbrace{\left(x^2 \cdot e \frac{-\lambda x}{-\lambda} \right)_0^\infty}_{0} - 2 \underbrace{\left[x \cdot \frac{e^{-\lambda x}}{\lambda^2} - 1 \frac{e^{-\lambda x}}{\lambda^2} \right]_0^\infty}_{0} \right]$$

$$2\lambda = \frac{2}{\lambda^2}$$

$\mu_2 = \mu_2' - (\mu_1)^2$

$$= \frac{2}{\lambda^2} - \left(\frac{1}{\lambda}\right)^2$$

$$= \frac{1}{\lambda^2} = \text{Variance}.$$

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Q9) Fit the data in second degree curve (parabola) by method of least square.

x	0	1	2	3	4
y	1	1.8	1.3	2.5	6.3

Sol $x = 5$

X	Y_0	X^2	X^3	X^4	XY_0	X^2Y_0
0	1	0	0	0	0	0
1	1.8	1	1	1	1.8	1.8
2	1.3	4	8	16	2.6	5.2
3	2.5	9	27	81	7.5	22.5
4	6.3	16	64	256	25.2	100.8
10	12.9	30	100	354	37.1	130.3

$Y_0 = a + bx + Cx^2$ — ①

Normal eq.n

$12.9 = 5a + 10b + 30c$ — ②

$37.1 = 10a + 30b + 100c$ — ③

$130.3 = 30a + 100b + 354c$ — ④

Multiply ② by ② and then subtract

$37.1 = 10a + 30b + 100c$

$25.8 = 10a + 20b + 60c$

$11.3 = 10b + 40c$ — ⑤

multiply ③ $\times 3$ and subtract ④

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$$130.3 = 30a + 100b + 354c$$
$$-111.8 = 30a + 90b + 300c$$
$$0.19 = 10b + 54c \quad — (6)$$

eqn 6 - 5

$$19 = 10b + 54c$$
$$-11.3 = 10b + 40c$$
$$7.7 = 14c$$
$$c = 0.55$$

Now as we know from eqn (6)

$$10b + 54(0.55) = 19$$
$$10b = 19 - 29.7$$
$$b = -1.07$$

Now, put b, c in (5)

$$12.9 = 5a - 10.7 + 16.5$$
$$5a = 12.9 - 5.8$$
$$5a = 7.1$$
$$a = 1.42$$

$$y_c = 1.42 - 1.07x + 0.55x^2$$

Req table.

x	0	1	2	3	4
y_c	1.42	0.9	1.48	3.16	5.94

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Q10) Prove that the angle between two regression lines is $\theta = \tan^{-1}\left\{\frac{(1-r)^2}{r} \cdot \frac{\sigma x \sigma x'}{\sigma x^2 + \sigma y^2}\right\}$ where r is the correlation coefficient.

Ans $\theta = \tan^{-1}\left\{\frac{m_1 - m_2}{1+m_1m_2}\right\}$

$m_1 = b_{yx} = r\frac{\sigma y}{\sigma x}.$

$m_2 = b_{xy} = \frac{\sigma y}{r\sigma x}.$

$$\theta = \tan^{-1}\left\{\frac{r\frac{\sigma y}{\sigma x} - \frac{\sigma y}{\sigma x}}{1 + \frac{r\sigma y}{\sigma x} \times \frac{\sigma y}{\sigma x}}\right\}$$

$$= \tan^{-1}\left\{\frac{\left(r - \frac{1}{\sigma}\right)\frac{\sigma y}{\sigma x}}{\frac{\sigma x^2 + \sigma y^2}{\sigma x^2}}\right\}$$

Ans

Tutorial Sheet - 1

Ques 1. A mild steel wire 5mm in diameter and 1m long. If the wire is subjected to an axial tensile load 10 KN find its extension of the rod. ($E = 200\,GPa$). ($2.55\,mm$)

Sol. Given:

$d = 5\,mm = 5 \times 10^{-3}\,m$

$l = 1\,m = 1 \times 10^{3}\,mm$

$P = 10\,KN = 10 \times 10^{3}\,N$

$E = 200\,GPa = 200 \times 10^{3}\,N/mm^2$

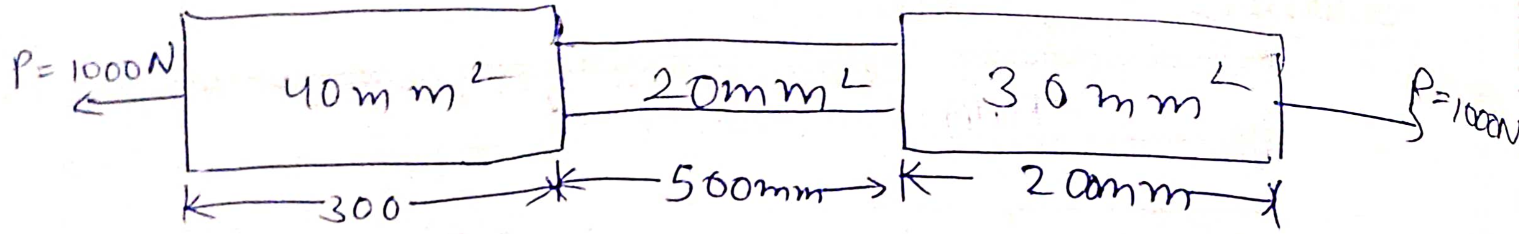
$\delta l = ?$

$$\delta l = \frac{P \cdot l}{A \cdot E}$$

$$= \frac{10 \times 10^{3} \times 10^{3}}{\frac{\pi}{4}(5)^2 \times 200 \times 10^{3}}$$

$$= 2.5464\,mm$$

Ques 2. A composite rod is 1000 mm long, its two ends are $40\,mm^2$ and $30\,mm^2$ in area and length 300 m and 200 mm respectively. The middle portion of the rod is $20\,mm^2$ in area and 500 mm long. If the load is 1000 N and its tensile load. find its elongation ($E = 200\,GPa$)



Sol. Given:- $P = 1000\,N$

$E = 200\,GPa = 200 \times 10^3\,N/mm^2$

$l_1 = 300\,mm$

$A_1 = 40\,mm^2$

$l_2 = 500\,mm$

$A_2 = 20\,mm^2$

$l_3 = 200\,mm$

$A_3 = 30\,mm^2$

$\delta l = ?$

we know ~~that~~ elongation of the component,

$$\delta l = \frac{P}{E}\left(\frac{l_1}{A_1} + \frac{l_2}{A_2} + \frac{l_3}{A_3}\right)$$

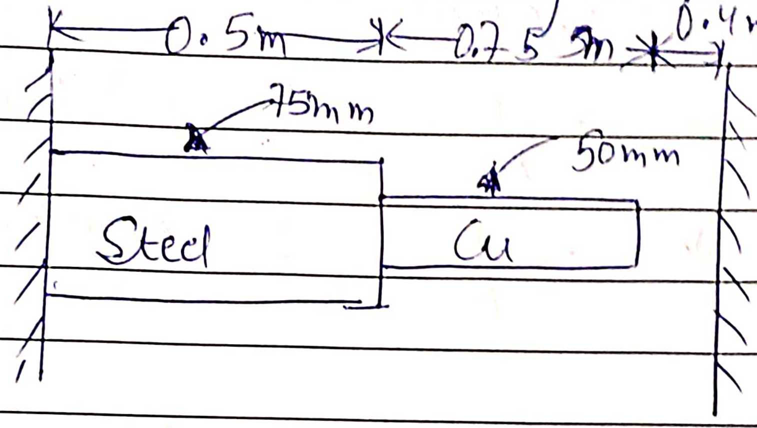
$$\delta l = \frac{\cancel{1000}}{200 \times \cancel{10^3}}\left(\frac{30\cancel{0}}{4\cancel{0}} + \frac{50\cancel{0}}{2\cancel{0}} + \frac{20\cancel{0}}{3\cancel{0}}\right)$$

$$= \frac{1}{200}\left(\frac{30}{4} + 25 + \frac{20}{3}\right)$$

$$= 0.195\,mm$$

Ques3 A rod consists of two parts that are made of steel and copper as show in fig. The elastic modulus and coefficient of ~~thermal~~ expansion for steel are 200 GPa and 19.7×10^{-6} per °C respectively and for copper 70 GPa and 21.6×10^{-6} per °C respectively. If the temperature of the rod is raised by 50°C ...

forces and stresses acting on the rod.



Given :- $l_s = 0.5m$

$l_c = 0.75m$

$d_s = 75mm$

$d_c = 50mm$

$E_s = 200 \times 10^3 N/mm^2$

$E_c = 70 \times 10^3 N/mm^2$

$\alpha_c = 21.6 \times 10^{-6}$ per °C

$\alpha_s = 11.7 \times 10^{-6}$ per °C

$T = 50°C$

thermal stress = $\cancel{\sigma_t = \alpha . T . E}$

$\cancel{\sigma_t = \alpha_s . T . E_s}$

$\cancel{= 11.7 \times 10^{-6} \times 50 \times 200 \times 10^3}$

$\cancel{= 11.7 \times 5 \times 2 \times 10^{-6} \times 10^3 \times 10^3}$

$\cancel{= 117}$

free expanses is

$\delta_T l = [\alpha_s T l_s] + [\alpha_c T l_c]$

$= 1.1025 mm$

But in fig free expansion $(\delta) = 0.4mm$

$\therefore \delta l = 1.1025 - 0.4 mm$

$= 0.7025 m$

Now,

$$\delta l = \left(\frac{Pl}{AE}\right)_s + \left(\frac{Pl}{AE}\right)_c$$

$P = 116.6 KN$

Now, $\sigma_s = \frac{P}{A_s} = 26.39 \, MPa$

$\sigma_c = \frac{P}{A_c} = 59.38 \, MPa$

Ques 1 A 10 mm diameter tensile specimen has 50 mm gauge length. The load corresponding to the 0.2% offset is 55 KN and the maximum load is 70 KN. Fracture occurs at 60 KN. The diameter after fracture is 8 mm and gauge length at fracture is 65 mm. Calculate the following properties of the material from the tension test.

(i) % Elongation
(ii) Reduction of Area (RA) %
(iii) Tensile strength or Ultimate tensile strength
(iv) Yield strength
(v) Fracture strength
(vi) If E = 200 GPa, the elastic recoverable strain at maximum load.
(vii) If the elongation at maximum load (the uniform elongation) is 20%, what is the plastic strain at maximum load

Sol. (i) % elogation

=> gauge length at fracture − gauge length

=> 65 mm − 50 mm

=> $\frac{15\,mm}{50\,mm} \times 100$

=> 30%

(iii) Reduction of area

(Initial dia.) $\frac{\pi}{4}(d_0)^2 = \frac{\pi}{4} \times (10)^2 = 78.53981634$

(Final dia.) $\frac{\pi}{4}\left(d_n\right)^2 = \frac{\pi}{4} \times (8)^2 = 50.26548246$

$$\% \text{ Red. of } A = \left(\frac{\pi/4\, d_0^2 - \pi/4\, d_n^2}{\pi/4\, d_0^2}\right) \times 100$$

$$= \frac{28.27433388}{78.53981634} \times 100$$

$$= 0.36 \times 100$$

$$= 36\%$$

(iv) Yield strength $= \frac{\text{Yield load}}{\text{C.s load}} = \frac{55 \times 10^3}{78.5391634}$

$C/s \text{ area} = \frac{\pi}{4}(d_0)^2 = \frac{\pi}{4}(10)^2 = 78.5391684$

$= 700 \text{ MPa}$

(iii) Ultimate stress $= \frac{\text{max}^m \text{ load}}{C/s \text{ area}} = \frac{70 \times 10^3}{78.5391634}$

$= 891 \text{ MPa}$

(v) Fracture strength $= \frac{\text{Fracture load}}{C/s \text{ area}} = \frac{60 \times 10^3}{78.5391634}$

$= 764 \text{ MPa}$

(vi) $E = 200 \times 10^3 \text{ N/mm}^2$

$\sigma = \frac{P}{A} = \frac{70 \times 10^3}{78.5391634}$

$\varepsilon = \frac{\sigma}{E} = \frac{P}{AE} = 4.45 \times 10^{-3}$

(vii) Elongation = 20%

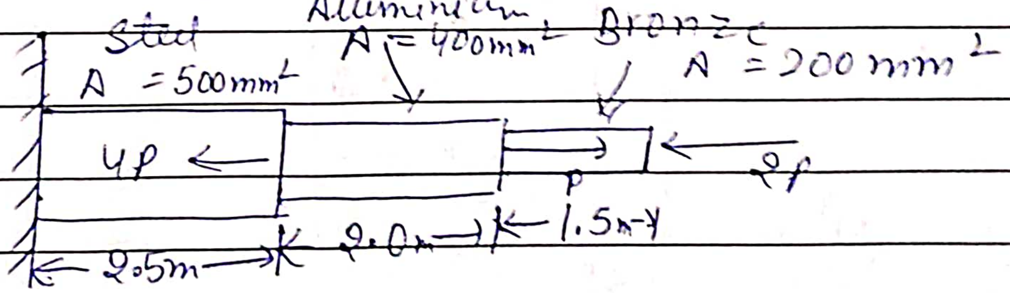
Strain = 0.20

$(\text{strain})_{max} = 0.0045$

Plastic strain = 0.20 − 0.0045

= 0.1955

Q5 An aluminium rod is rigidly attached between a steel rod and a bronze rod as shown in figure, Axial loads are applied at the position indicated. Find the maximum value of P that will not exceed a stress in steel of 140 MPa in aluminium of 90 MPa or in bronze of 100 MPa. (10 KN)



Sol Given:- $A_S = 500\,mm^2$

$A_A = 400\,mm^2$

$A_B = 200\,mm^2$

$\sigma_S = 140 \times 10^3\ N/mm^2$

$\sigma_A = 90 \times 10^3\ N/mm^2$

$\sigma_B = 100 \times 10^3\ N/mm^2$

Maximum load → 2P

$$\text{Stress} = \frac{\text{Maxm load}}{\text{C/S area}}$$

For Brone

$$\sigma_B = \frac{2P}{A_B}$$

$$\frac{100 \times 10^3 \times \overset{1}{\cancel{200}}}{\cancel{2}} = P$$

$$\boxed{P_{max} = 10 KN}$$

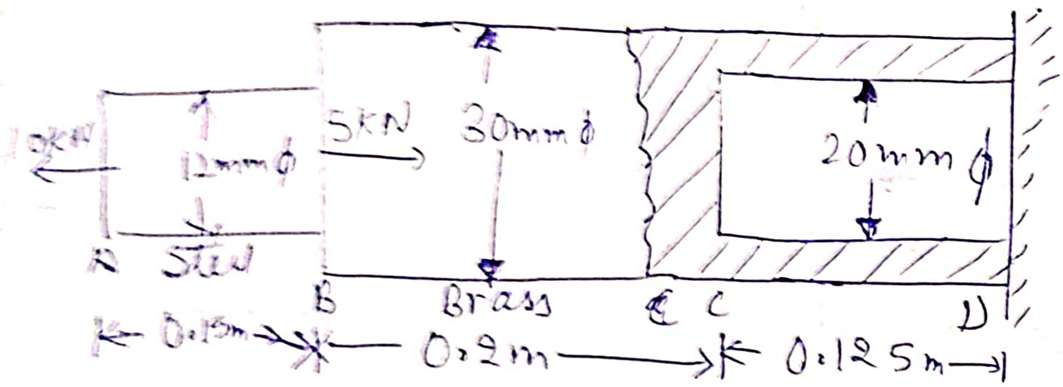
Ques 6 The diameters of the brass and the steel segments of the axially loaded bar shown in fig are 30 mm and 12 mm respectively. The diameter of the hallow section of the brass segment is 20 mm.

Determine :- (i) The max^m normal stress in the steel and the brass

(ii) The displacement of free end

Take $E_s = 210$ GN/m^2

$E_b = 105$ GN/m^2



Sol.

Normal stress for Brass $= \frac{P_{max}}{\text{C/S area}}$

$$= \frac{5 \times 10^3 \text{ N}}{\frac{\pi}{4} \times (30 \times 10^{-3})^2}$$

$$= \frac{5 \times 10^3}{\frac{\pi}{4} \times 900 \times 10^{-6}}$$

Normal Stress = 7.07 MN/m²

Normal stress for steel = $\frac{P_{max}}{C/S \text{ area}}$

$= \frac{10 \times 10^3}{\frac{\pi}{4} \times (12 \times 10^{-3})^2}$

$= 88.42 \, MN/m^2$

(ii) The displacement of the free end

$\delta l = (\delta l_s)_{AB} + (\delta l_B)_{BC} + (\delta l_B)_{CD}$

$= \frac{88.42 \times 0.15}{210 \times 10^9 \times 10^{-6}} + \frac{7.07 \times 0.2}{105 \times 10^9 \times 10^{-6}} + \frac{12.73 \times 0.125}{105 \times 10^9 \times 10^{-6}}$

$= 9.178 \times 10^{-5} \, m = 0.09178 \, mm$

Ques 7 A copper rod 6 cm in diameter is placed within a steel tub, 8 cm external diameter and 6 cm internal diameter of exactly of same length. The two pieces are rigidly fixed together by two transverse pins 20 mm in diameter, one at each end passing through both the rods and tubes.

Calculate the stresses produced in copper rod, steel tube and the pins if the temperature of the combination is raised by 50°C.

Take $E_S = 210 GN/m^2$

$E_C = 105 GN/m^2$

$\alpha_S = 1.15 \times 10^{-05}/°C$

$\alpha_C = 1.7 \times 10^{-05}/°C$

Sol. Given :-

$d_c = 6 \times 10^{-2} m$

$A_C = \frac{\pi}{4} d_c^2 = \frac{\pi}{4} \times (0.06)^2 = 0.0009\pi$

$= 2.827 \times 10^{-3} m^2$

$D_s = 8 cm = 0.08 m$

$d_s = 6 cm = 0.06 m$

$A_s = \frac{\pi}{4}(D_s^2 - d_s^2) = \frac{\pi}{4}(0.08^2 - 0.06^2)$

$= \cancel{0.4998739} = 0.7 \times 10^{-4}\pi$

$\Delta T = 50°C$

$\sigma_C = ?$, $\sigma_S = ?$, $\sigma_P = ?$

$E_s = 210 GN/m^2 = 210 \times 10^9 N/m^2$

$E_C = 105 GN/m^2 = 105 \times 10^9 N/m^2$

$\alpha_s = 1.15 \times 10^{-5}/°C$

$\alpha_C = 1.7 \times 10^{-5}/°C$

load on each bar is equal

$\sigma_S A_S = \sigma_c A_c \quad [\because \sigma = \frac{P}{A}]$

$\sigma_c \times 0.0009\pi = \sigma_s A_s$

$= \sigma_s \cancel{0.1591 \times \pi}\ 0.7 \times 10^{-4} \times \pi$

$\sigma_C \times 0.0009\pi = \sigma_s \times 0.7 \times 10^{-4} \times \pi$

$$\sigma_c \times 9\times10^{-4} \times \pi = \sigma_s \times 7\times10^{-4}\times\pi$$

$$\sigma_s = 1.2857\,\sigma_c \quad -(1)$$

Expansion of steel rod = Expansion of copper tube

$$\alpha_s(\Delta T)\cancel{L} + \frac{\sigma_s}{E_s}\times\cancel{L} = \alpha_c(\Delta T)\cancel{L} - \frac{\sigma_c}{E_c}\times\cancel{L}$$

$$\alpha_s(\Delta T) + \frac{\sigma_s}{E_s} = \alpha_c(\Delta T) - \frac{\sigma_c}{E_c}$$

$$\Rightarrow 1.15\times10^{-5}\times50 + \frac{1.2857}{210\times10^9}\sigma_c = 1.7\times10^{-5}\times50 - \frac{\sigma_c}{105\times10^9}$$

$$\Rightarrow \frac{\sigma_c}{10^9}\left[\frac{1.2857}{210} + \frac{1}{105}\right] = 1.7\times10^{-5}\times50 - 1.15\times10^{-5}\times50$$

$$\Rightarrow \frac{\sigma_c}{10^9}\left[\frac{1.2857\times105+210}{210\times105}\right] = 27.5\times10^{-5}$$

$$\boxed{\sigma_c = 17.58\text{ MPa}}$$

$$\sigma_s = 1.2857\times17.58\text{ MPa}$$

$$\boxed{\sigma_s = 22.6\text{ MPa}}$$

for transvers pin

$$A_c\times\sigma_c = P$$

$$P = 17.58\text{ MPa}\times 2.827\times10^{-3}$$

$$\sigma_{pin} = \frac{P}{A_p}$$

$$= \frac{17.58\text{ MPa}\times2.827\times10^{-3}}{2\times\frac{\pi}{4}\times(20\times10^{-3})^2}$$

$$\boxed{\sigma_{pin} = \cancel{8a.}\ 79\text{ MPa}}$$

Ques 6 A steel wire 2m long and 3mm in diameter is extended by 0.75mm when a weight W is suspended from the wire. If the same weight is suspended from a brass wire, 2.5m long and 2mm in diameter, it is elongated by 4.64 mm.

Determine $E_B = ?$

If $E_s = 2.0 \times 10^5 \ N/mm^2$

Sol. Given :- $l_s = 2m$ $\qquad P = W$

$d_s = 3 \times 10^{-3} m$

$\delta l_s = 0.75 \times 10^{-3} m$

$P = W$

$l_b = 2.5 m$

$d_b = 2 \times 10^{-3} m$

$\delta l_b = 4.64 \times 10^{-3} m$

$A_s = \frac{\pi}{4} (d_s)^2 = \frac{\pi}{4} (3 \times 10^{-3})^2 = 7.0685834\!\!\!\;71 \times 10^{-6}$

$A_b = \frac{\pi}{4} (d_b)^2 = \frac{\pi}{4} (2 \times 10^{-3})^2 = 3.141592654 \times 10^{-6}$

For steel

$\sigma_s = E_s \, \varepsilon$

$\frac{P}{A E_s} = \varepsilon$

$W = \frac{\delta l \times A \times E_s}{l}$

$= \frac{0.75 \times 10^{-3} \times 2.25 \pi \times 10^{-6} \times 2 \times 10^{5}}{2}$

$= 1.6875 \pi \times 10^{-4}$

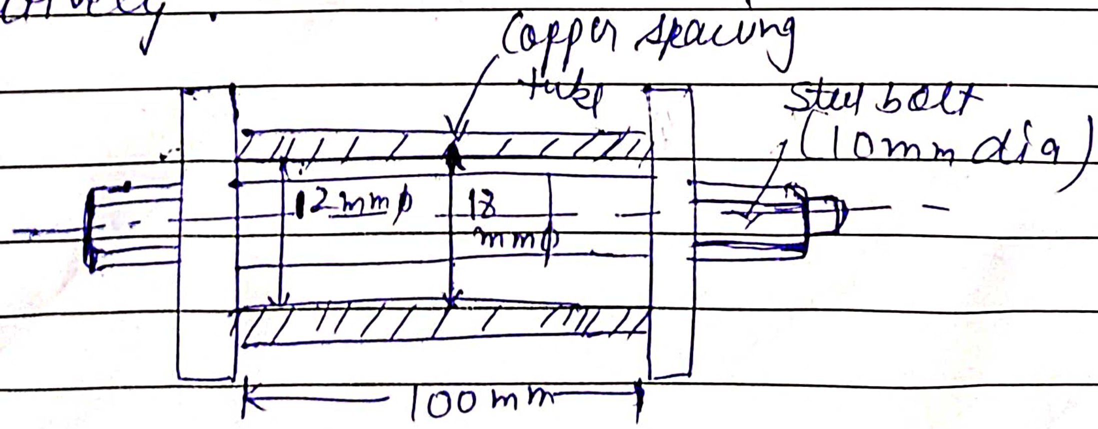
For brass

$\frac{P}{A_b E_b} = \varepsilon$

$E_B = \frac{1.6875 \times 10^{-4} \times 10^{6} \times 2.5}{4.64 \times 10^{-3}}$

$E_B = 0.909 \times 10^5 \ N/mm^2$

Q) The steel bolt shown in fig has a Thread pitch of 1.6 mm if the nut is initially tightened up by hand so as to cause no stress in the copper spacing tube, Calculate the stresses induced in the tube and in the bolt if a spanner is then used to turn the nut through 90°. Take Ec and Es as 100 GPa and 209 GPa respectively.



Course Name : Mechanics of solid
Course Code 3ME4-07
Faculty Name Dr. Mukesh Didwania
Unit No: 1-2
Tutorial Sheet No.: 1
Submission Date

Sol. Stressed induced in the tube and the bolt σ_c, σ_s:-

$$A_s = \frac{\pi}{4} \times \left(\frac{10}{1000}\right)^2 = 7.584 \times 10^{-5} m^2$$

$$A_c = \frac{\pi}{4} \times \left[\left(\frac{18}{1000}\right)^2 - \left(\frac{12}{1000}\right)^2\right] = 14.14 \times 10^{-5} m^2$$

Tensile forces on steel bolt, P_s = compressive force in copper tube, $P_c = P$, also increase in length of bolt + decrease in length of tube = axial displacement of nut

$$(\delta l)_s + (\delta l)_c = 1.6 \times \frac{90}{360} = 0.4 mm = 0.4 \times 10^{-3} m$$

or $$\frac{Pl}{A_s E_s} + \frac{Pl}{A_c E_c} = 0.4 \times 10^{-3} \quad (l_s = l_c = l)$$

or $$P \times \left(\frac{100}{1000}\right)\left[\frac{1}{7.854 \times 10^{-5} \times 209 \times 10^9} + \frac{1}{14.14 \times 10^{-5} \times 100 \times 10^9}\right]$$

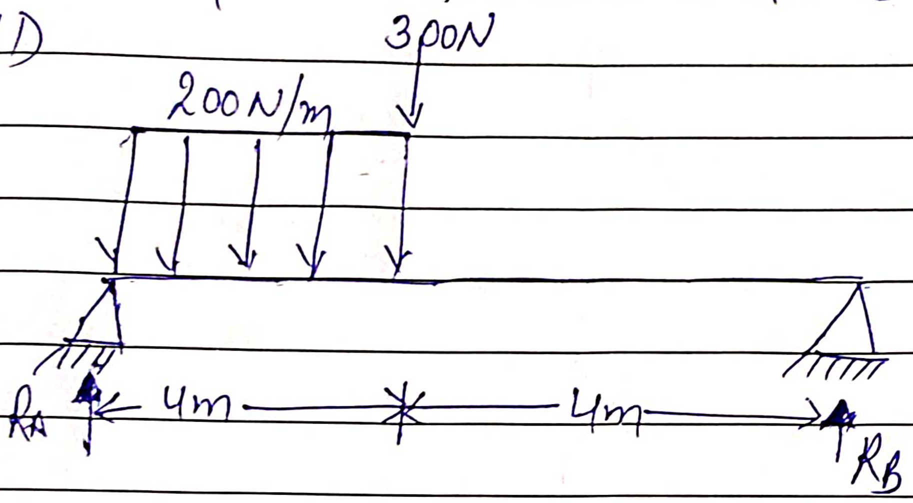
$$= 0.4 \times 10^{-3} m$$

$$P = 3038.6 N$$

$$\therefore \frac{P}{A_s} = 386.88 MPa \quad \text{and} \quad \frac{P}{A_c} = 214.89 MPa$$

Assignment No. 2

Que 1 A loaded beam as shown below. Draw its SFD and BMD



Sol. for equilibrium condition

$\Sigma F_x = 0$

$\Sigma F_y = 0$ $\quad \uparrow (+ve), \; (-ve) \downarrow$

$R_A + R_B - 800 - 300N = 0$

$\boxed{R_A + R_B = 3800\,N}$ — (1)

$\Sigma M = 0$ $\quad$ Clockwise $(-ve)$

$R_B \times 8 - 300 \times 4 - 800 \times 2 = 0$ $\quad$ Anti $(+ve)$

$\boxed{R_B = 1700\,N}$ — (2)

By eq (1) & (2) we get

$R_A + 1700 = 3800$

$R_A = 3800 - 1700$

$\boxed{R_A = 2100\,N}$

SFD :- $B \rightarrow A$ $\quad$ upward −ve $\quad$ Downward +ve

$SF_B = -1700$

$SF_C = 3000 - 1700 = +1300$

$SF_x = 1300 + 200 \times x \Rightarrow$ ($x = 0$, at A and $x = 4$ at C)

$SF_A = 1300 + 200 \times 4$
$\Rightarrow 2100\,(+ve)\,N$

POORNIMA

Bending Moment $(B \to A)$ (Anticlockwise +ve)

$M_B = 0$

$M_C = -R_B \times 4 = -(-1700 \times 4) \Rightarrow 6800\ Nm\ (+ve)$

$M_X = 1700(x+4) - 300x - 200 \times \frac{x}{2} \times x$

at C $x = 0$

$M_C = 6800\ Nm$

at A $x = 4$

$M_A = 6800 - 1300 \times 4 - 100 \times 16$

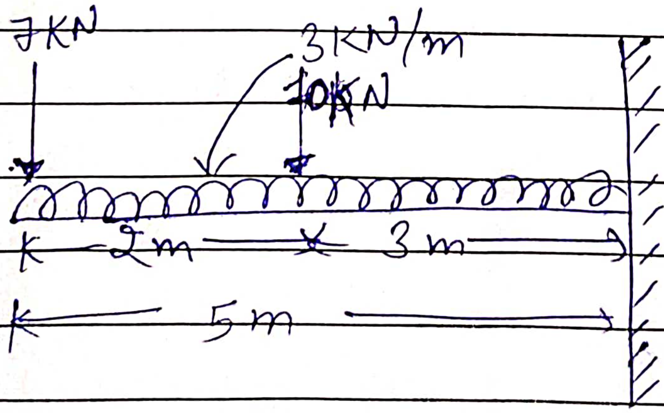
$= 6800 - 5200 - 1600$

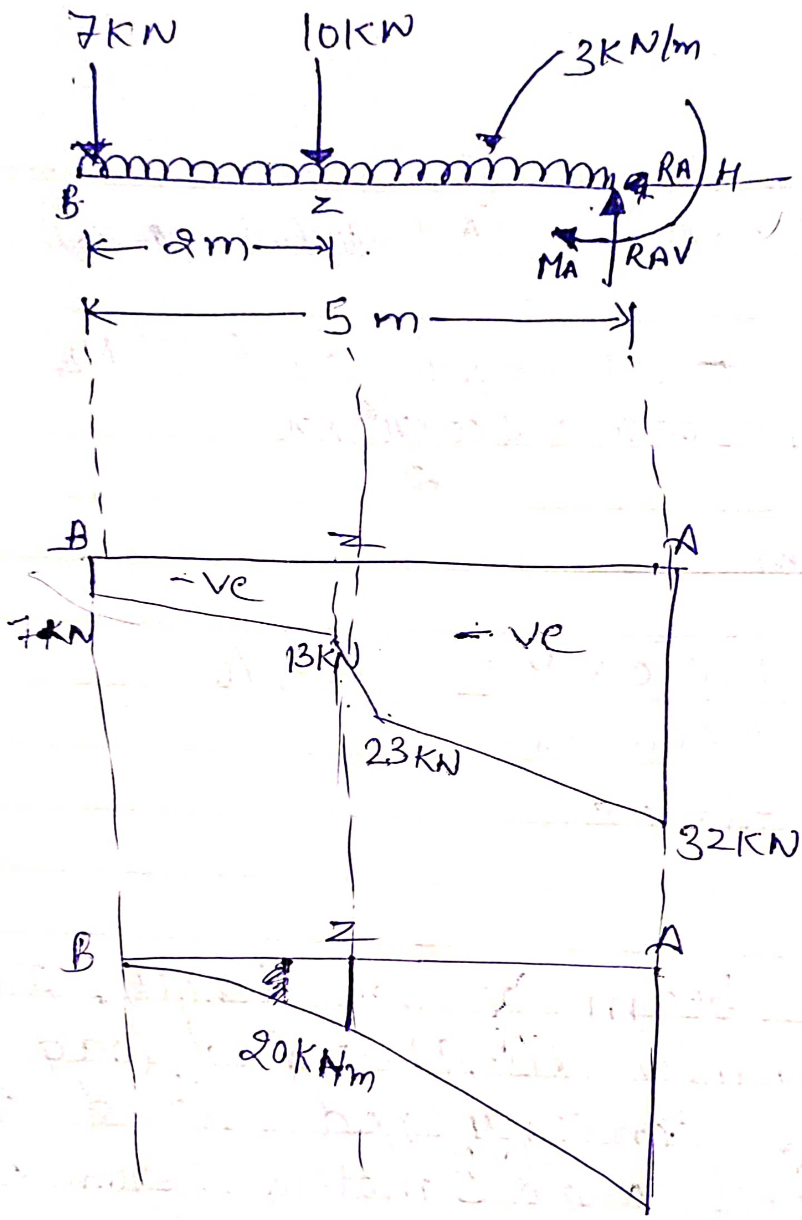
$= 6800 - 6800$

$M_A = 0$

Que Consider a cantilever beam of 5m length. It carries a uniformly distributed load 3KN/m and a concentrated load of 7KN at the free end and 10KN at 3 meters from the fixed end. Draw SFD & BMD.

Sol




7KN
10KN
3KN/m
RA
H
B
Z
2m
MA
RAV
5 m
B
Z
A
-ve
7KN
13KN
-ve
23KN
32KN
B
Z
A
20KNm

for equilibrium condition

$\Sigma F_X = 0$

$\boxed{R_{AH} = 0}$

$\Sigma F_Y = 0$ upward +ve

$R_{AV} - 3 \times 5 - 10 - 7 = 0$

$R_{AV} = 15 + 10 + 7$

$\boxed{R_{AV} = 32\,N}$

$\Sigma M = 0$ Anti CW − ve

$M_A - 3 \times 5 \times 2.5 - 10 \times 3 - 7 \times 5 = 0$

$M_A = 35 + 30 + 37.5$

$\boxed{M_A = 102.5}$

Shear force ↑ = −ve ↓ = +ve

$SF_A = -R_{AV} = -32$

$SF_Z = -32 + 3 \times 3 = -23\,KN$

$= -23 + 10 = -13\,KN$

$SF_B = -13 + 3 \times 2 = -7$

$= -7 + 7 = 0$

for Bending Moment for portion BZ, take section at distance x from the free end B.

$BM = -7x - 3 \times \frac{x \times x}{2}$

at point B

$x = 0$

$BM = 0$

at point Z

$x = 2$ $BM = -7 \times 2 - 3 \times 2 \times \frac{2}{2} \Rightarrow -14 - 6 \Rightarrow -20$

For portion ZA, again consider a section at distance x form the end B.

$$BM = -7\times x - 13(x-2) - 3\times x\times\frac{x}{2}$$

At $x=2$

$$BM = -7\times2 - 13(2-2) - 3\times2\times\cancel{\frac{2}{2}}$$
$$= -14 - 6$$
$$= -20 \text{ KN m}$$

At $x=3$

$$BM = -7\times3 - 13(3-2) - 3\times3\times\frac{3}{2}$$
$$= -21 - 13 - \frac{27}{2}$$
$$= -21 - 13 - 13.5$$
$$= -47.5 \text{ KN m}$$

Ques 3 A water ~~tank~~ main of 120 cm, internal diameter and 12mm thick is running full. If the bending stress is not too exceed 560 kg/cm² find the greatest span on which the pipe may be freely supported steel and water weigh is 7680 kg/m³ and 1000 kg/m³ respectively

Sol. Given

$d = 120$ cm $= 1,2000$ mm

$D = 12000 + 12$

$= 12012$ mm $= 121.2$ cm

$\sigma_b = 560 \text{ kg/cm}^2$

$$\therefore \frac{M}{I} = \frac{\sigma_b}{y} = \frac{E}{R}$$

$$\frac{M}{I} = \frac{\sigma_b}{y}$$

$$\therefore I = \frac{\pi}{64}(D^2 - d^2)$$
$$= 836600 \text{ cm}^4$$

$$M = \frac{\sigma}{y} I = \frac{560}{61.2}$$