ADVANCED ENGG. MATHS 3ed sem(IT)
TUTORIAL SHEET–II
UNIT-II (PROBABILITY DISTRIBUTION)

<u>NOTE:-</u> Attempt all questions.

Q.(1) Find mean, variance and standard deviation of Binomial Distribution.

Q.(2) In a certain factory turning out razor blades there is a small chance of 0.002 for any blade to be Defective .the blades are supplied in a packet of 10. Using Poisson distribution, find the Number of packets containing no defective, one defective, and two defective blades respectively in a consignment of 10,000 packets.

Q.(3) The distribution of weekly wages for 500 workers in a factory is approximately normal with The mean and standard deviation of Rs.75 and Rs.15 respectively .find the number of workers Who receive weekly wages (i) more than Rs.90 (ii) less than Rs.46. (given $p(0\le z< 1) = 0.3413$,$P(0<z<2) = 0.4772$)

Q.(4) Fit a Binomial distribution to the following data :-

x	0	1	2	3	4	5
f	2	14	20	34	22	8

Q.(5) Find the Karl Pearson coefficient of correlation from the following data.

x	25	27	30	35	33	28	36
y	19	22	27	28	30	23	28

Q.(6) The ranks of 10 students in two subjects mathematics and physics are given below:

Ranks in mathematics x	5	2	9	8	1	10	3	4	6	7
Ranks in physics y	10	5	1	3	8	6	2	7	9	4

Calculate Spearman's Rank Correlation coefficient

Q.(7) Calculate the coefficient of correlation and obtain the line of regression for the following data.

x	1	2	3	4	5	6	7	8	9
y	9	8	10	12	11	13	14	16	15

Q.(8) Define Exponential distribution? Find the mean and variance of the distribution.

Q.(9) Fit the data in second degree curve (parabola) by method of least square.

x	0	1	2	3	4
y	1	1.8	1.3	2.5	6.3

Q.10 Prove that the angle between two regression lines is $\theta = tan^{-1}\{(\frac{1-r^2}{r})\frac{\sigma x\sigma x}{\sigma_x{}^2+\sigma_y{}^2}\}$ where r is the Correlation coefficient.

22/10/22

Unit II – Binomial Distribution

Q1) Find mean, variance and standard deviation of binomial distribution.

Mean

Let x be any discrete random variable

$\mu_1' = E(x) = \Sigma x P(x)$

$P(r) = {}^nC_r p^r q^{n-r}$ $[r = 1, 2, 3, \ldots n]$

$\mu_1' = \sum_{r=1}^{n} r \cdot {}^nC_r p^r q^{n-r}$.

$\mu_1' = {}^nC_1 pq^{n-1} + 2\,{}^nC_2 p^2 q^{n-2} + 3\,{}^nC_3 p^2 q^{n-3} + \ldots$

$= npq^{n-1} + 2(n)(n-1)p^2 q^{n-2} + 3(n)(n-1)(n-2)p^3 q^{n-3} + \ldots$

$\mu_1' = npq^{n-1}\left[1 + (n-1)\frac{p}{q} + (n-1)(n-2)\frac{p^2}{q^2}\right]$

OR

$\mu_1' = E(x) = \sum_{r=0}^{\infty} r \cdot {}^nC_r p^r q^{n-r}$

$= \sum_{r=0}^{\infty} r \frac{\lfloor n}{\lfloor r \lfloor n-r} p^r q^{n-r}$.

$\sum_{r=0}^{\infty} \frac{r \lfloor n}{r \lfloor r-1 \lfloor n-r} p^r q^{n-r}$

$\sum_{r=0}^{\infty} \frac{n \lfloor n-1}{\lfloor r-1 \lfloor n-1-r+1} \times p^{r-1+1} q^{n-1-r+1}$

$= \sum_{r=1}^{\infty} np \frac{\lfloor n-1}{\lfloor r-1 \lfloor n-1-r+1} p^{r-1} q^{n-1-r+1}$

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$= \sum_{r=1}^{\infty} np \; {}^{n-1}C_{r-1} \; p^{r-1} \; q^{n-1-r+1}$

$= np(p+q)^{n-1} = np$

So $n = \mu_1' = np.$

Variance,

$\sigma^2 = \mu_2 = E(X^2) - [E(X)]^2$

$\mu_2' - (\mu_1)^2$ — ①

$\mu_2' = E(X^2) = \sum_{r=0}^{\infty} r^2 p(r)$

$\sum_{r=0}^{\infty} r^2 \; {}^nC_r p^r q^{n-r}.$

$= \sum_{r=0}^{\infty} [r(r-1)+r] \; {}^nC_r p^r q^{n-r}.$

$\sum_{r=0}^{\infty} r(r-1) \; {}^nC_r p^r q^{n-r} + \sum_{r=0}^{\infty} r \; {}^nC_r p^r q^{n-r}$

$= \sum_{r=0}^{\infty} r(r-1) \frac{\lfloor n}{\lfloor r \lfloor n-r} p^r q^{n-r} + np$

$= \sum_{r=0}^{\infty} \frac{r(r-1) \lfloor n}{r(r-1) \lfloor r-2 \lfloor n-r} p^r q^{n-r} + np$

$\sum_{r=2}^{\infty} \frac{n(n-1) \lfloor n-2}{\lfloor r-2 \lfloor n-2-r+2} p^{r-2+2} \cdot q^{n-2-r+2} + np.$

$= n(n-1)p^2 (p+q)^{n-2} + np.$

$\mu_2' = n(n-1)p^2 + np$

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$\mu_2' = n^2p^2 - np^2 + np$ — (2)

$\sigma^2 = n^2p^2 - np^2 + np - n^2p^2$

$\sigma^2 = np - np^2$

$\sigma^2 = np(1-p) = npq$

$\sigma^2 = npq$ — variance

$SD = \sigma = \sqrt{\text{variance}}$

so standard deviation $= \sqrt{npq}$

Q2) In a certain factory turning out razor blades there is a small chance of 0.002 for any blade to be defective the blades are supplied in a packet of 10. using poison distribution, find the number of packets containing no defective, one defective, and two defective blades respectively in a consignment of 10,000 packets

Ans given $p = 0.002$

$n = 10$

$M = 10,000$

given condition

(i) No defective

(ii) one defective

(iii) ~~two~~ defective

Mean $= m = np = 10 \times 0.002 = 0.02$

we have by poison's distribution

$$p(x=r) = \frac{e^{-m} m^r}{\lfloor r} \; ; \; r = 0,1,2, \ldots$$

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(i) $p(x=0) = \dfrac{e^{-0.02} \times (0.02)^0}{\lfloor 0} = 0.9801$

So No of packets having no defective blades out of 10,000 packets

$f(x=0) = N \times P(x=0)$

$= 10,000 \times 0.9801 = 9801$ packets

(ii) $P(x=1) = \dfrac{e^{-0.02} \times (0.02)^1}{\lfloor 1} = 0.0196$

So No of packets having one defective blade out of 10,000 packets

$= 10,000 \times 0.0196 = 196B$.

(iii) $P(x=2) = \dfrac{e^{-0.02} (0.02)^2}{\lfloor 2} = 1.96 \times 10^{-4}$.

So, the no of ~~blades~~ packets having two defective blades out of 10,000 packets.

Q3) The distribution of weekly wages for 500 workers in a factory is approximately normal with the mean and standard deviation of Rs 75 and Rs 15 respectively. Find the number of workers who receive weekly wages

(i) more than Rs 90

(ii) less than Rs 46 (given $p(0 \le z < 1) = 0.3413$, $P(0 < z < 2) = 0.4772$)

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Sol given : $\mu = 75$

$\sigma = 15$

$N = 500$

$$Z = \frac{x - \mu}{\sigma}$$

(i) more than Rs 90

So $x = 90$ $= P(x > 90)$

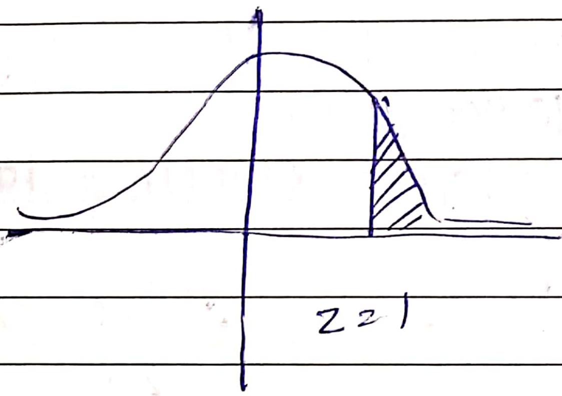
$$\frac{90 - 75}{15} = 1$$

or $P(z > 1)$

$P(0 < z < \infty) - P(0 < z < 1)$

$0.5 - 0.3413$

$= 0.1587$



So the total no of workers who received weekly wages more than Rs 90

$= 0.157 \times 500$

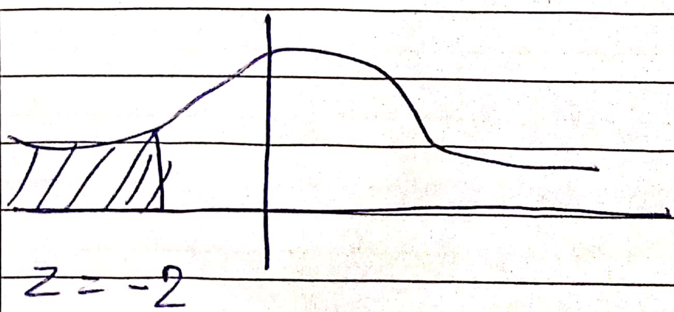
$= 79.35$ Ans :

~~79~~ 79 Ans .

(ii) Less than Rs 46

$P(x < 46)$

$$Z = \frac{46 - 75}{15} = -1.93 = -2$$



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$P(Z < -2) = P(-\infty < Z < 0) - P(-1 < Z < 0)$

$= 0.5 - 0.4772$

$= 0.228.$

So, the total numbers of workers who received weekly wages less than Rs 46

$= 0.0228 \times 500$

$= 11.4 \simeq 11$

Q4. Fit a Binomial distribution to the following data.

x	0	1	2	3	4	5
f	2	14	20	34	22	8

Ans Here we have f

So $m = \frac{\Sigma x.f}{\Sigma f} = \frac{284}{100} = 2.84$

x	f	x.f
0	2	0
1	14	14
2	20	40
3	34	102
4	22	88
5	8	40

$\Sigma f = 100 \quad \Sigma x.f = 284$

We have

$m = np$

$2.84 = 5(p)$

$p = 0.568.$

$q = 1-p = 0.432$

So $= (0.568 + 0.432)^5$

Theoritical frequency

$f(x) = N \times (p+q)^n$

$= 100 \times (0.568 + 0.432)^5$

$= 100 \times 1 + 100 \times 1^2 + 100 \times 1^3 + 100 \times 1^4 + 100 \times 1^5 + 100 \times 1^6 = 600$ Ans

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Q5) Find the Karl Pearson coefficient of correlation from the following data.

x	25	27	30	35	33	28	36
y	19	22	27	28	30	23	28

Ans We have Karl Pearson Coefficient

for $r(x,y) = \frac{\Sigma (x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\Sigma (x_i - \bar{x})} \cdot \sqrt{(\Sigma y_i - \bar{y})^2}}$

$A = (x_i - \bar{x})$; $B = (y_i - \bar{y})$

$r(x,y) = \frac{\Sigma AB}{\sqrt{\Sigma A^2} \sqrt{\Sigma B^2}}$

x	y	$x_i - \bar{x}$	$y_i - \bar{y}$	$(x_i - \bar{x})(y_i - \bar{y})$	$(x_i - \bar{x})^2$	$(y - \bar{y})^2$
25	19	-5.57	-6.28	34.97	31.02	39.43
27	22	-3.57	-3.28	11.70	12.74	10.75
30	27	-0.57	1.72	0.98	0.32	3.06
35	28	4.43	2.72	12.04	19.62	7.39
33	30	2.43	4.72	11.46	5.90	22.27
28	23	-2.57	-2.28	5.85	6.60	5.198
36	28	5.43	2.72	14.76	29.48	7.39
				$\Sigma AB - 89.8$	$\Sigma A = 105.68$	$\Sigma B - 95.4$

$\bar{X} = \frac{25 + 27 + 30 + 35 + 33 + 28 + 36}{7} = 30.58$

$\bar{Y} = \frac{19 + 22 + 27 + 28 + 30 + 28 + 28}{7} = 25.29$

$r(x,y) = \frac{89.8}{105.68 \times 95.48} = 8.89 \times 10^{-03}$

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Q6) The ranks of 10 students in two subjects mathematics and physics are given below

Ranks in mathematics x	5	2	9	8	1	10	3	4	6	7
Ranks in physics y	10	5	1	3	8	6	2	7	9	4

Calculate Spearman's Rank Correlation Coefficient

Ans $n = 10$ no of students

given

R_x	R_y	$d = R_x - R_y$	d^2
5	10	-5	25
2	5	-3	9
9	1	8	64
8	3	5	25
1	8	-7	49
10	6	4	16
3	2	1	1
4	7	-3	9
6	9	-3	9
7	4	3	9

Σd^2 216

We have by spearson's Rank correlation formula

$$\rho(x,y) = 1 - \frac{6 \Sigma d^2}{n(n^2-1)} = 1 - \frac{6 \times 216}{10 \times 99}$$

$$= -0.3091$$

Hence the rank in subject maths and physics are -ve correlated.

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Q7. Calculate the coefficient of correlation and obtain the line of regression for the following data:

x	1	2	3	4	5	6	7	8	9
y	9	8	10	12	11	13	14	16	15

Ans

X	Y	X-5	Y-12	AB	A^2	B^2
1	8	-4	-4	16	16	16
2	9	-3	-3	9	9	9
3	10	-2	-2	4	4	4
4	12	-1	0	0	1	0
5	11	0	-1	0	0	1
6	13	1	1	1	1	1
7	14	2	2	4	4	4
8	16	3	4	12	9	16
9	15	4	3	12	16	9
				$\Sigma AB = 58$	$\Sigma A^2 = 60$	$\Sigma B^2 = 60$

$$r(x,y) = \frac{\Sigma(x_i-\bar{x})(y_i-\bar{y})}{\sqrt{\Sigma(x_i-\bar{x})^2 \Sigma(y_i-\bar{y})^2}}$$

$(x_i-\bar{x}) = A$; $(y_i-\bar{y}) = B$

$$r(x,y) = \frac{\Sigma AB}{\sqrt{A^2 \Sigma B^2}} = \frac{58}{60 \times 60} = 0.96621$$

So $r(x,y) = 1$

$\sigma x = 0.860$

$\sigma y = 0.60$

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Equation of regression line :-

y on x

$$y - \bar{y} = r \frac{\sigma y}{\sigma x} (x - \bar{x}) \quad - (1)$$

$\cancel{y - 12 = 0}$

$$y - 12 = 0.966 \times 1 \ (x - 5)$$

$$y - 12 = 0.966 \ x \ - 4.83$$

$$y = 0.966x + 7.17$$

x on y

$$(x - \bar{x}) = r \left(\frac{\sigma x}{\sigma y}\right) (y - \bar{y})$$

$$x - 5 = 0.96 \times \frac{0.860}{0.860} (x - 12) = 0.96y - 0.96 \times 12$$

$$x = 0.96y - 6.52 \text{ Ans.}$$

Q2 Define Exponential distribution? Find the mean and variance of the distribution.

Ans Exponential Distribution

A continuous random variable X is said to follow an exponents distribution also called as negative exponential distribution with parameter $\lambda > 0$ if its probability density function is given by

$$f(x) = \begin{cases} \lambda e^{-\lambda x} & , \ x \geq 0 \\ 0 & ; \ \text{otherwise} \end{cases}$$

The above given is Pdf as $f(x) > 0$ and

$$\int_0^\infty f(x) dx = \lambda \int_0^\infty e^{-\lambda x} dx = \lambda \left(\frac{e^{-\lambda x}}{-\lambda}\right)_0^\infty = 1$$

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Mean and Variance

Mean $\mu_1' = \int_0^\infty x \lambda e^{-\lambda x}\, dx$.

$$= \lambda \left[x \cdot \frac{e^{-\lambda x}}{-\lambda} - 1 \cdot \frac{e^{-\lambda x}}{\lambda^2} \right]_0^\infty$$

$$= \lambda \times \frac{1}{\lambda^2} = \frac{1}{\lambda}$$

So $\mu_1' = \frac{1}{\lambda}$

Variance $= \mu_2 = \mu_2' - (\mu_1')^2$

$$\mu_2' = \int_0^\infty x^2 \lambda e^{-\lambda x}\, dx$$

$$= \lambda \int_0^\infty x^2 e^{-\lambda x}\, dx$$

$$= \lambda \left[\left(x^2 \cdot \frac{e^{-\lambda x}}{-\lambda} \right)_0^\infty - 2\int_0^\infty x \frac{e^{-\lambda x}}{-\lambda}\, dx \right]_0^\infty$$

$$= \lambda \left[\underbrace{\left(x^2 \cdot \frac{e^{-\lambda x}}{-\lambda} \right)_0^\infty}_{0} - 2 \left[\underbrace{x \frac{e^{-\lambda x}}{\lambda^2}}_{0} - 1 \frac{e^{-\lambda x}}{\lambda^2} \right]_0^\infty \right]$$

$$2\lambda = \frac{2}{\lambda^2}$$

$$\mu_2 = \mu_2' - (\mu_1)^2$$

$$= \frac{2}{\lambda^2} - \left(\frac{1}{\lambda}\right)^2$$

$$= \frac{1}{\lambda^2} = \text{Variance}.$$

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Q9) Fit the data in second degree curve (parabola) by method of least square.

x	0	1	2	3	4
y	1	1.8	1.3	2.5	6.3

Sol $x = 5$

X	Y_0	x^2	x^3	x^4	xY_0	x^2Y_0
0	1	0	0	0	0	0
1	1.8	1	1	1	1.8	1.8
2	1.3	4	8	16	2.6	5.2
3	2.5	9	27	81	7.5	22.5
4	6.3	16	64	256	25.2	100.8
10	12.9	30	100	354	37.1	130.3

$Y_0 = a + bx + cx^2$ — ①

Normal eq.-n

$12.9 = 5a + 10b + 30c$ — ②

$37.1 = 10a + 30b + 100c$ — ③

$130.3 = 30a + 100b + 354c$ — ④

Multiply ② by ② and then subtract

$37.1 = 10a + 30b + 100c$

$25.8 = 10a + 20b + 60c$

$11.3 = 10b + 40c$ — ⑤

multiply ③ $\times 3$ and subtract ④

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$120.3 = 30a + 100b + 354c$
$-111.8 = 30a + 90b + 300c$
$0.19 = 10b + 54c$ — (6)

eqn 6 − 5
$19 = 10b + 54c$
$-11.3 = 10b + 40c$
$7.7 = 14c$
$c = 0.55$

Now as we know from eqn (6)
$10b + 54(0.55) = 19$
$10b = 19 - 29.7$
$b = -1.07$

Now, put b, c in (5)
$12.9 = 5a - 10.7 + 16.5$
$5a = 12.9 - 5.8$
$5a = 7.1$
$a = 1.42$

$y_c = 1.42 - 1.07x + 0.55x^2$

Req table.

x	0	1	2	3	4
y_c	1.42	0.9	1.48	3.16	5.94

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Q10) Prove that the angle between two regression lines is $\theta = \tan^{-1}\left\{\frac{(1-r)^2}{r} \cdot \frac{\sigma x \sigma x'}{\sigma x^2 + \sigma y^2}\right\}$ where r is the correlation coefficient.

Ans $\theta = \tan^{-1}\left\{\frac{m_1 - m_2}{1 + m_1 m_2}\right\}$

$m_1 = b_{yx} = r\frac{\sigma y}{\sigma x}$.

$m_2 = b_{xy} = \frac{\sigma y}{r \sigma x}$.

$$\theta = \tan^{-1}\left\{\frac{r\frac{\sigma y}{\sigma x} - \frac{\sigma y}{\sigma x}}{1 + r\frac{\sigma y}{\sigma x} \times \frac{\sigma y}{\sigma x}}\right\}$$

$$= \tan^{-1}\left\{\frac{\left(r - \frac{1}{\sigma}\right)\frac{\sigma y}{\sigma x}}{\frac{\sigma x^2 + \sigma y^2}{\sigma x^2}}\right\}$$ Ans