

Tutorial Sheet - 1

Q1. A mild steel wire 5mm in diameter and 1m long. If the wire is subjected to an axial tensile load 10 kN find its extension of the rod. ($E = 200 \text{ GPa}$). (2.55 mm)

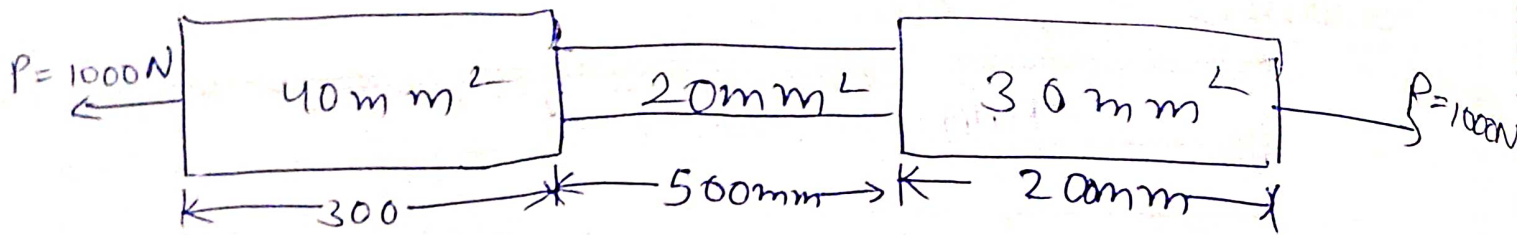
Given: $d = 5 \text{ mm} = 5 \times 10^{-3} \text{ m}$
 $L = 1 \text{ m} = 1 \times 10^3 \text{ mm}$
 $P = 10 \text{ kN} = 10 \times 10^3 \text{ N}$
 $E = 200 \text{ GPa} = 200 \times 10^3 \text{ N/mm}^2$
 $\delta L = ?$

$$\delta L = \frac{P \cdot L}{A \cdot E}$$

$$= \frac{10 \times 10^3 \times 10^3}{\frac{\pi (5)^2}{4} \times 200 \times 10^3}$$

$$= 2.5464 \text{ mm}$$

Ques 2. A composite rod is 1000 mm long, its two ends are 40 mm² and 30 mm² in area and length 300 mm and 200 mm respectively. The middle portion of the rod is 20 mm² in area and 500 mm long. If the load is 100 kN and its tensile load. find its elongation ($E = 200 \text{ GPa}$)



Sol.

Given:- $P = 1000 \text{ N}$

$$E = 200 \text{ GPa} = 200 \times 10^3 \text{ N/mm}^2$$

$$l_1 = 300 \text{ mm}$$

$$A_1 = 40 \text{ mm}^2$$

$$l_2 = 500 \text{ mm}$$

$$A_2 = 20 \text{ mm}^2$$

$$l_3 = 200 \text{ mm}$$

$$A_3 = 30 \text{ mm}^2$$

$$\Delta l = ?$$

We know that elongation of the component,

$$\Delta l = \frac{P}{E} \left(\frac{l_1}{A_1} + \frac{l_2}{A_2} + \frac{l_3}{A_3} \right)$$

$$\Delta l = \frac{1000}{200 \times 10^3} \left(\frac{300}{40} + \frac{500}{20} + \frac{200}{30} \right)$$

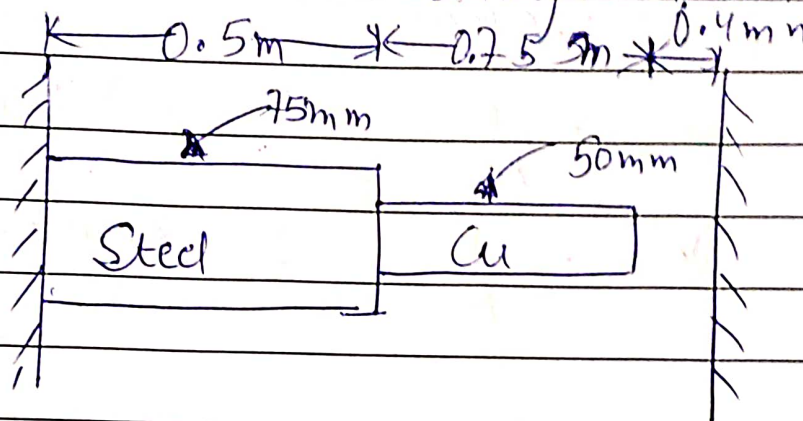
$$= \frac{1}{200} \left(\frac{30}{4} + 25 + \frac{20}{3} \right)$$

$$= 0.195 \text{ mm}$$

Ques 3 A rod consists of two parts that are made of steel and copper as shown in fig. The elastic modulus and coefficient of thermal expansion for steel are 200 GPa and $11.7 \times 10^{-6} \text{ per } ^\circ\text{C}$ respectively and for copper 70 GPa and $21.6 \times 10^{-6} \text{ per } ^\circ\text{C}$ respectively. If the temperature of the rod is raised by 50°C find the change in length.

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forces and stresses acting on the rod.



Given :- $l_s = 0.5m$

$$l_c = 0.75m$$

$$d_s = 75mm$$

$$d_c = 50mm$$

$$E_s = 200 \times 10^3 N/mm^2$$

$$E_c = 70 \times 10^3 N/mm^2$$

$$\alpha_c = 21.6 \times 10^{-6} \text{ per } ^\circ C$$

$$\alpha_s = 11.7 \times 10^{-6} \text{ per } ^\circ C$$

$$T = 50^\circ C$$

Thermal strain = $\Delta l = \alpha \cdot T \cdot l$

$$\Delta l = \alpha_s \cdot T \cdot l_s$$

$$= 11.7 \times 10^{-6} \times 50 \times 200 \times 10^3$$

$$= 11.7 \times 5 \times 2 \times 10^{-6} \times 10^3 \times 10^3$$

$$= 117$$

Free expansion is

$$\Delta l = [\alpha_s \cdot T \cdot l_s] + [\alpha_c \cdot T \cdot l_c]$$

$$= 1.1025mm$$

But in fig free expansion $(\Delta l) = 0.4mm$

$$\therefore \Delta l = 1.1025 - 0.4mm$$

$$= 0.7025 \text{ m}$$

Now,

$$\delta l = \left(\frac{Pl}{AE} \right)_S + \left(\frac{Pl}{AE} \right)_C$$

$$P = 116.6 \text{ kN}$$

$$\text{Now, } \delta_S = \frac{P}{AE} = 26.39 \text{ mm}$$

$$\delta_C = \frac{P}{A_C} = 59.38 \text{ mm}$$

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Q. 4 A 10 mm diameter tensile specimen has 50 mm gauge length. The load corresponding to the 0.2% offset is 55 kN and the maximum load is 70 kN. Fracture occurs at 60 kN.

The diameter after fracture is 8 mm and gauge length at fracture is 65 mm. Calculate the following properties of the material from the tension test.

- (i) % Elongation
- (ii) Reduction of Area (RA) %
- (iii) Tensile strength or Ultimate tensile strength
- (iv) Yield strength
- (v) Fracture strength
- (vi) If $E = 200 \text{ GPa}$, the elastic recoverable strain at maximum load.
- (vii) If the elongation at maximum load (the uniform elongation) is 20%, what is the plastic strain at maximum load

Sol. 1) % elongation

=> gauge length at fracture - gauge length

$$=) \frac{65 \text{ mm} - 50 \text{ mm}}$$

$$=) \frac{15 \text{ mm}}{50 \text{ mm}} \times 100$$

$$=) 30\%$$

(ii) Reduction of area

$$\left(\begin{array}{l} \text{Initial} \\ \text{dia.} \end{array} \right) \quad \frac{\pi}{4} (d_o)^2 = \frac{\pi}{4} \times (10)^2 = 78.53981634$$

$$\left(\begin{array}{l} \text{Final} \\ \text{dia.} \end{array} \right) \quad \frac{\pi}{4} (d_n)^2 = \frac{\pi}{4} \times (8)^2 = 50.26548246$$

$$\% \text{ Red. of } A = \left(\frac{\frac{\pi}{4} d_o^2 - \frac{\pi}{4} d_n^2}{\frac{\pi}{4} d_o^2} \right) \times 100$$

$$= \frac{28.27433388}{78.53981634} \times 100$$

$$= 0.36 \times 100$$

$$= 36\%$$

$$(iv) \quad \text{Yield strength} = \frac{\text{Yield load}}{\text{C.S. load}} = \frac{55 \times 10^3}{78.53981634}$$

$$\text{C/S area} = \frac{\pi}{4} (d_o)^2 = \frac{\pi}{4} (10)^2 = 78.53981634$$
$$= 700 \text{ MPa}$$

$$(iii) \quad \text{Ultimate stress} = \frac{\text{max}^m \text{load}}{\text{C/S area}} = \frac{70 \times 10^3}{78.53981634}$$
$$= 891 \text{ MPa}$$

$$(v) \quad \text{Fracture strength} = \frac{\text{Fracture load}}{\text{C/S area}} = \frac{60 \times 10^3}{78.53981634}$$
$$= 764 \text{ MPa}$$

$$(vi) \quad E = 200 \times 10^3 \text{ N/mm}^2$$
$$\sigma = \frac{P}{A} = \frac{70 \times 10^3}{78.53981634}$$

$$\epsilon = \frac{\sigma}{E} = \frac{P}{AE} = 4.45 \times 10^{-3}$$

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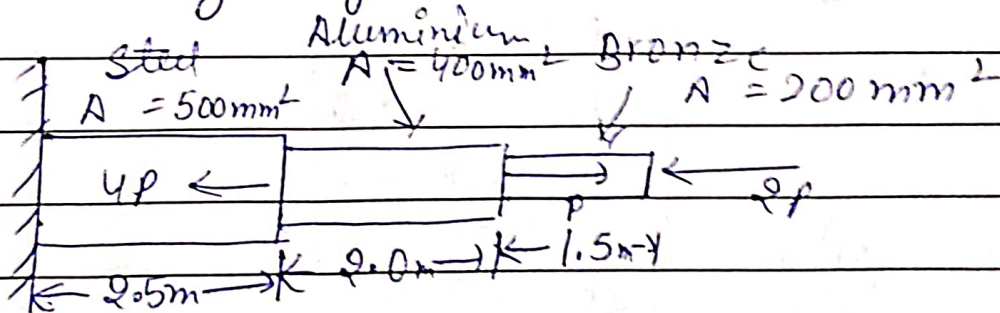
(vi) Elongation = 20%

Strain = 0.20

(strain)_{max} = 0.0045

Elastic strain = $0.20 - 0.0045$
 = 0.1955

Q. An aluminium rod is rigidly attached between a steel rod and a bronze rod as shown in figure. Axial loads are applied at the positions indicated. find the maximum value of P that will not exceed a stress in steel of 140 MPa in aluminium of 90 MPa or in bronze of 100 MPa . (1015N)



Given: $A_s = 500 \text{ mm}^2$

$A_A = 400 \text{ mm}^2$

$A_B = 200 \text{ mm}^2$

$\sigma_s = 140 \times 10^3 \text{ N/mm}^2$

$\sigma_A = 90 \times 10^3 \text{ N/mm}^2$

$\sigma_B = 100 \times 10^3 \text{ N/mm}^2$

Maximum load $\rightarrow 2P$

$$\text{Stress} = \frac{\text{max load}}{\text{c/s area}}$$

for bronze

$$\sigma_b = \frac{2P}{A_b}$$

$$\frac{100 \times 10^3 \times 200}{\cancel{2}} = P$$

$$P_{\max} = 10 \text{ kN}$$

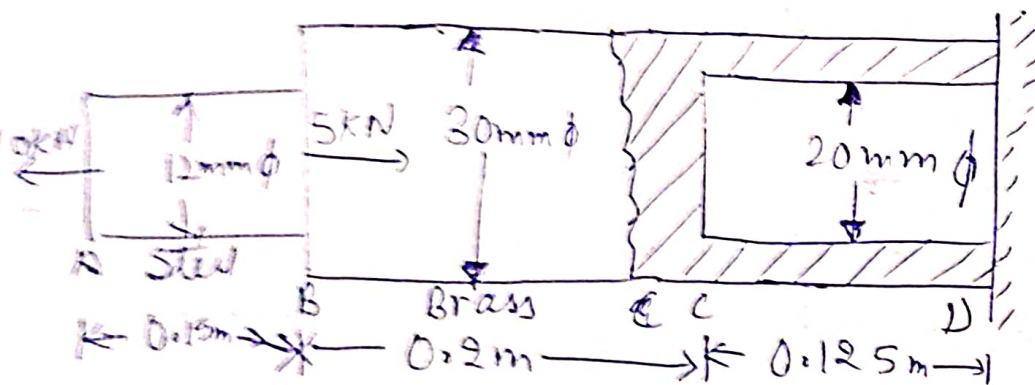
Ques 5 The diameters of the brass and the steel segments of the axially loaded bar shown in fig are 30mm and 12mm respectively. The diameter of the hollow section of the brass segment is 20mm.

Determine :- (i) The max^m normal stress in the steel and the brass

(ii) The displacement of free end

Take $E_s = 210 \text{ GN/m}^2$

$E_b = 105 \text{ GN/m}^2$



Sol

Normal stress = $\frac{P_{\max}}{\text{c/s area}}$

For Brass

$$= \frac{5 \times 10^3 \text{ N}}{\frac{\pi}{4} \times (30 \times 10^{-3})^2}$$

$$= \frac{5 \times 10^3}{\frac{\pi}{4} \times 900 \times 10^{-6}}$$

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$$\text{Normal Stress} = 7.07 \text{ MN/m}^2$$

$$\begin{aligned} \text{Normal stress for steel} &= \frac{P_{\max}}{\text{C/S area}} \\ &= \frac{10 \times 10^3}{\frac{\pi}{4} \times (12 \times 10^{-3})^2} \\ &= 88.42 \text{ MN/m}^2 \end{aligned}$$

(ii) The displacement of the free end

$$\begin{aligned} \delta l &= (\delta l_s)_{AB} + (\delta l_s)_{BC} + (\delta l_s)_{CD} \\ &= \frac{88.42 \times 0.15}{210 \times 10^9 \times 10^{-6}} + \frac{7.07 \times 0.2}{105 \times 10^9 \times 10^{-6}} + \frac{12.73 \times 0.125}{105 \times 10^9 \times 10^{-6}} \\ &= 9.178 \times 10^{-5} \text{ m} = 0.09178 \text{ mm} \end{aligned}$$

Ques 7 A copper rod 6 cm in diameter is placed within a steel tube, 8 cm external diameter and 6 cm internal diameter of exactly of same length. The two pieces are rigidly fixed together by two transverse pins 20 mm in diameter, one at each end passing through both the rods and tubes.

Calculate the stresses induced in the copper tube and the pins if the temperature of the combination is raised by 50°C .

Take $E_S = 210 \text{ GN/m}^2$

$E_C = 105 \text{ GN/m}^2$

$\alpha_S = 1.15 \times 10^{-5}/^\circ\text{C}$

$\alpha_C = 1.7 \times 10^{-5}/^\circ\text{C}$

sol. Given :- $d_c = 6 \times 10^{-2} \text{ m}$

$$A_C = \frac{\pi}{4} d_c^2 = \frac{\pi}{4} \times (0.06)^2 = 0.0009\pi$$

$$= 2.827 \times 10^{-3} \text{ m}^2$$

$D_S = 8 \text{ cm} = 0.08 \text{ m}$

$d_S = 6 \text{ cm} = 0.06 \text{ m}$

$$A_S = \frac{\pi}{4} (D_S^2 - d_S^2) = \frac{\pi}{4} (0.08^2 - 0.06^2)$$

$$= 0.0014 \pi \text{ m}^2 = 0.7 \times 10^{-4} \pi$$

$\Delta T = 50^\circ\text{C}$

$\sigma_C = ?$, $\sigma_S = ?$, $\sigma_P = ?$

$E_S = 210 \text{ GN/m}^2 = 210 \times 10^9 \text{ N/m}^2$

$E_C = 105 \text{ GN/m}^2 = 105 \times 10^9 \text{ N/m}^2$

$\alpha_S = 1.15 \times 10^{-5}/^\circ\text{C}$

$\alpha_C = 1.7 \times 10^{-5}/^\circ\text{C}$

load on each bar is equal

$\sigma_S A_S = \sigma_C A_C$ $\left[\because \sigma = \frac{P}{A} \right]$

$\sigma_C \times 0.0009\pi = \sigma_S A_S$

$$= \sigma_S \times 0.0014\pi$$

$\sigma_C \times 0.0009\pi = \sigma_S \times 0.0014\pi$

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$$\sigma_c \times 9 \times 10^{-4} \times \pi = \sigma_s \times 7 \times 10^{-4} \times \pi$$

$$\sigma_s = 1.2857 \sigma_c \quad \text{--- (1)}$$

Expansion of steel rod = Expansion of copper tube

$$\alpha_s (\Delta T) L + \frac{\sigma_s}{E_s} \times L = \alpha_c (\Delta T) L - \frac{\sigma_c}{E_c} \times L$$

$$\alpha_s (\Delta T) + \frac{\sigma_s}{E_s} = \alpha_c (\Delta T) - \frac{\sigma_c}{E_c}$$

$$\Rightarrow 1.15 \times 10^{-5} \times 50 + \frac{1.2857 \sigma_c}{210 \times 10^9} = 1.7 \times 10^{-5} \times 50 - \frac{\sigma_c}{105 \times 10^9}$$

$$\Rightarrow \frac{\sigma_c}{10^9} \left[\frac{1.2857}{210} + \frac{1}{105} \right] = 1.7 \times 10^{-5} \times 50 - 1.15 \times 10^{-5} \times 50$$

$$\Rightarrow \frac{\sigma_c}{10^9} \left[\frac{1.2857 \times 105 + 210}{210 \times 105} \right] = 27.5 \times 10^{-5}$$

$$\sigma_c = 17.58 \text{ MPa}$$

$$\sigma_s = 1.2857 \times 17.58 \text{ MPa}$$

$$\sigma_s = 22.6 \text{ MPa}$$

for transverse pin

$$A_c \sigma_c = P$$

$$P = 17.58 \text{ MPa} \times 2.827 \times 10^{-3}$$

$$\sigma_{pin} = \frac{P}{A_p}$$

$$= \frac{17.58 \text{ MPa} \times 2.827 \times 10^{-3}}{2 \times \frac{\pi}{4} \times (20 \times 10^{-3})^2}$$

$$\sigma_{pin} = 79 \text{ MPa}$$

Ques 6 A steel wire 2m long and 3mm in diameter is extended by 0.75mm when a weight W is suspended from the wire. If the same weight is suspended from a brass wire, 2.5m long and 2mm in diameter, it is elongated by 4.64 mm.

Determine $E_B = ?$

$$\text{If } E_s = 2.0 \times 10^5 \text{ N/mm}^2$$

Sol. Given: $l_s = 2\text{m}$ $P = W$

$$A_s = \frac{\pi}{4} (d_s)^2 = \frac{\pi}{4} (3 \times 10^{-3})^2$$

$$= 7.068583471 \times 10^{-6} \text{ m}^2$$

$$d_s = 3 \times 10^{-3} \text{ m}$$

$$\delta l_s = 0.75 \times 10^{-3} \text{ m}$$

$$P = W$$

$$A_b = \frac{\pi}{4} (d_b)^2 = \frac{\pi}{4} (2 \times 10^{-3})^2$$

$$= 3.141592654 \times 10^{-6} \text{ m}^2$$

$$l_b = 2.5 \text{ m}$$

$$d_b = 2 \times 10^{-3} \text{ m}$$

$$\delta l_b = 4.64 \times 10^{-3} \text{ m}$$

For steel

$$\sigma_s = E_s \epsilon$$

$$\frac{P}{A E_s} = \epsilon$$

$$W = \frac{\delta l}{l} \times A \times E_s$$

$$= \frac{0.75 \times 10^{-3}}{2} \times 2.25 \pi \times 10^6 \times 2 \times 10^{-6}$$

$$= 1.6875 \pi \times 10^{-4}$$

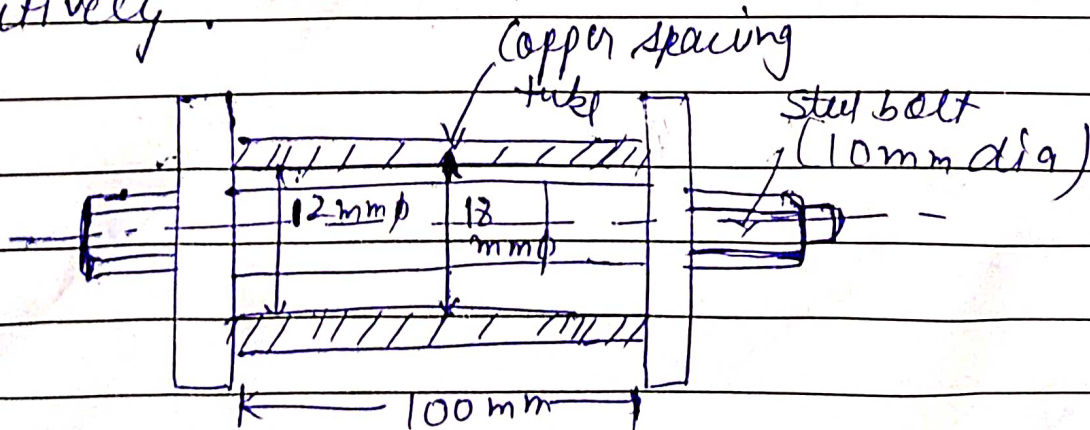
For brass

$$\frac{P}{A_b E_b} = \epsilon$$

$$E_b = \frac{1.6875 \times 10^{-4} \times 10^6 \times 2.5}{4.64 \times 10^{-3}}$$

$$E_b = 0.907 \times 10^5 \text{ N/mm}^2$$

Q. The steel bolt shown in fig has a thread pitch of 1.6 mm. If the nut is initially tightened up by hand so as to cause no stress in the copper spacing tube, calculate the stresses induced in the tube and in the bolt if a spanner is then used to turn the nut through 90° . Take E_c and E_s as 100 GPa and 209 GPa respectively.



1 Sol. Stressed induced in the tube and the bolt
 $\sigma_c, \sigma_s:-$

$$A_s = \frac{\pi}{4} \times \left(\frac{10}{1000} \right)^2 = 7.854 \times 10^{-5} \text{ m}^2$$

$$A_c = \frac{\pi}{4} \times \left[\left(\frac{18}{1000} \right)^2 - \left(\frac{12}{1000} \right)^2 \right] = 14.14 \times 10^{-5} \text{ m}^2$$

Tensile forces on steel bolt, $p_s = \text{compressive}$
 force in copper tube, $p_c = p$, also increase
 in length of bolt + decrease in length of
 tube = axial displacement of nut

$$(\delta l)_s + (\delta l)_c = \frac{1.6 \times 90}{360} = 0.4 \text{ mm} = 0.4 \times 10^{-3} \text{ m}$$

$$\text{or } \frac{p l}{A_s E_s} + \frac{p l}{A_c E_c} = 0.4 \times 10^{-3} \quad (l_s = l_c = l)$$

$$\text{or } p \times \left(\frac{100}{1000} \right) \left[\frac{1}{7.854 \times 10^{-5} \times 209 \times 10^9} + \frac{1}{14.14 \times 10^{-5} \times 100 \times 10^9} \right] = 0.4 \times 10^{-3} \text{ m}$$

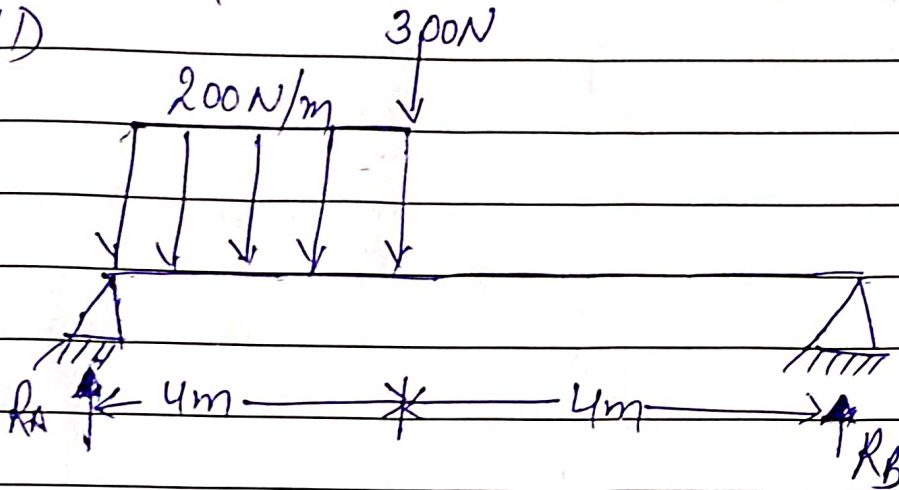
$$p = 3038.6 \text{ N}$$

$$\therefore \frac{p}{A_s} = 386.88 \text{ MPa} \quad \text{and} \quad \frac{p}{A_c} = 214.89 \text{ MPa}$$

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Assignment No. 2

Ques 1 A loaded beam as shown below. Draw its SFD and BMD



sol. for equilibrium condition

$$\sum F_x = 0$$

$$\sum F_y = 0$$

↑ (+ve), (-ve) ↓

$$R_A + R_B - 800 - 300 \text{ N} = 0$$

$$R_A + R_B = 1100 \text{ N} \quad (1)$$

$$\sum M = 0$$

Clockwise (-ve)
 Anti (+ve)

$$R_B \times 8 - 300 \times 4 - 800 \times 2 = 0$$

$$R_B = 1700 \text{ N} \quad (2)$$

By eq (1) & (2) we get

$$R_A + 1700 = 1100$$

$$R_A = 1100 - 1700$$

$$R_A = -600 \text{ N}$$

SFD :- B → A upward -ve Downward +ve

$$SFB = -1700$$

$$SFC = 3000 - 1700 = +1300$$

$$SF_x = 1300 + 200 \times x \Rightarrow (x=0 \text{ at A}$$

$$SFA = 1300 + 200 \times 4$$

$$\Rightarrow 2100 (+ve) \text{ N}$$

and $x=4$ at C)

Bending Moment ($B \rightarrow A$) (Anticlockwise $\rightarrow$ ve)

$$-M_B = 0$$

$$M_C = +R_B \times 4 = +(-1700 \times 4) = 6800 \text{ Nm (+ve)}$$

$$M_x = 1700(x+4) - 300x - 200 \times \frac{x^2}{2} \times x$$

$$\text{at } C \quad x = 0$$

2

$$M_C = 6800 \text{ Nm}$$

$$\text{at } A \quad x = 4$$

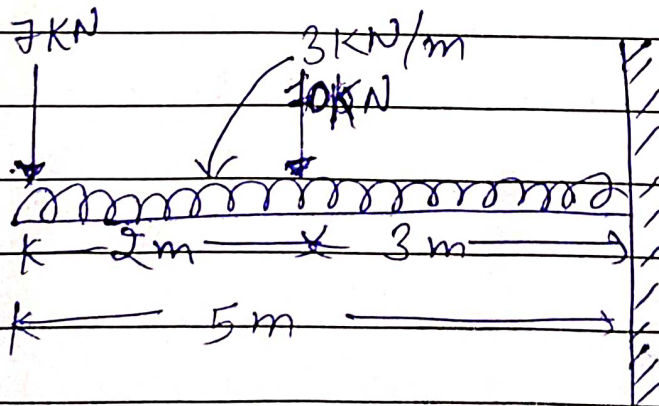
$$M_A = 6800 - 1300 \times 4 - 100 \times 16$$

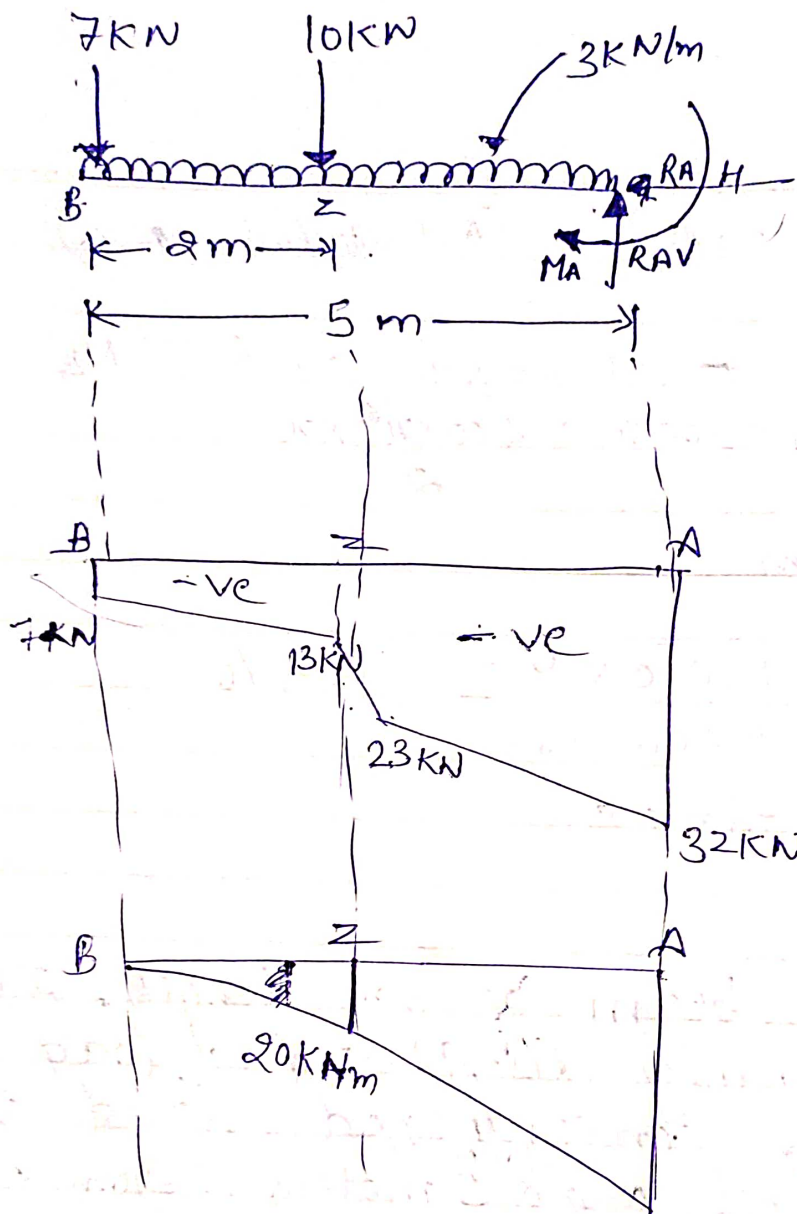
$$= 6800 - 5200 - 1600$$

$$= 6800 - 6800$$

$$M_A = 0$$

Ques. Consider a cantilever beam of 5m length. It carries a uniformly distributed load 3kN/m and a concentrated load of 7kN at the free end and 10kN at 3 meters from the fixed end. Draw SFD & BMD.





for equilibrium condition

$$\sum F_x = 0$$

$$R_{AH} = 0$$

$$\sum F_y = 0$$

upward +ve

$$R_{AV} - 3 \times 5 - 10 - 7 = 0$$

$$R_{AV} = 15 + 10 + 7$$

$$R_{AV} = 32 \text{ N}$$

$$\sum M = 0$$

Anti CW -ve

$$M_A - 3 \times 5 \times 2.5 - 10 \times 3 - 7 \times 5 = 0$$

$$M_A = 35 + 30 + 32.5$$

$$M_A = 102.5$$

Shear force

$\uparrow = -ve$

$\downarrow = +ve$

$$S_{FA} = -R_{AV} = -32$$

$$S_{FZ} = -32 + 3 \times 3 = -23 \text{ KN}$$

$$= -23 + 10 = -13 \text{ KN}$$

$$S_{FB} = -13 + 3 \times 2 = -7$$

$$= -7 + 7 = 0$$

for Bending Moment for portion ABZ, take section at distance x from the free end B.

$$BM = -7x - 3 \times x \times x$$

at point B

$$x = 0$$

$$BM = 0$$

at point Z

$$x = 2$$

$$BM = -7 \times 2 - 3 \times 2 \times \frac{2}{2} = -14 - 6 = -20$$

for portion ZA, again consider a section at distance x from the end B.

$$BM = -7 \times x - 13(x-2) - 3 \times x \times \frac{x}{2}$$

At $x=2$

$$\begin{aligned} BM &= -7 \times 2 - 13(2-2) - 3 \times 2 \times \frac{2}{2} \\ &= -14 - 6 \\ &= -20 \text{ KNm} \end{aligned}$$

At $x=3$

$$\begin{aligned} BM &= -7 \times 3 - 13(3-2) - 3 \times 3 \times \frac{3}{2} \\ &= -21 - 13 - \frac{27}{2} \\ &= -21 - 13 - 13.5 \\ &= -47.5 \text{ KNm} \end{aligned}$$

Ques 3 A water tank main of 120 cm, internal diameter and 12 mm thick is running full. If the bending stress is not to exceed 560 kg/cm² find the greatest span on which the pipe may be freely supported steel and water weight is 7680 kg/m³ and 1000 kg/m³ respectively.

Sol.

Given

$$d = 120 \text{ cm} = 1200 \text{ mm}$$

$$D = 12000 + 12$$

$$= 12012 \text{ mm} = 121.2 \text{ cm}$$

$$\sigma_b = 560 \text{ kg/cm}^2$$

$$\therefore \frac{M}{I} = \frac{\sigma_b}{y} = \frac{E}{R}$$

$$\frac{M}{I} = \frac{\sigma_b}{y}$$

$$\therefore I = \frac{\pi (D^4 - d^4)}{64}$$

$$= 836600 \text{ cm}^4$$

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$$M = \frac{\sigma}{y} I = \frac{560}{61.2}$$